Jan 18 lec covers section 1-4 and 2-4 along with material on conservation laws which is not in the book (references are given below).

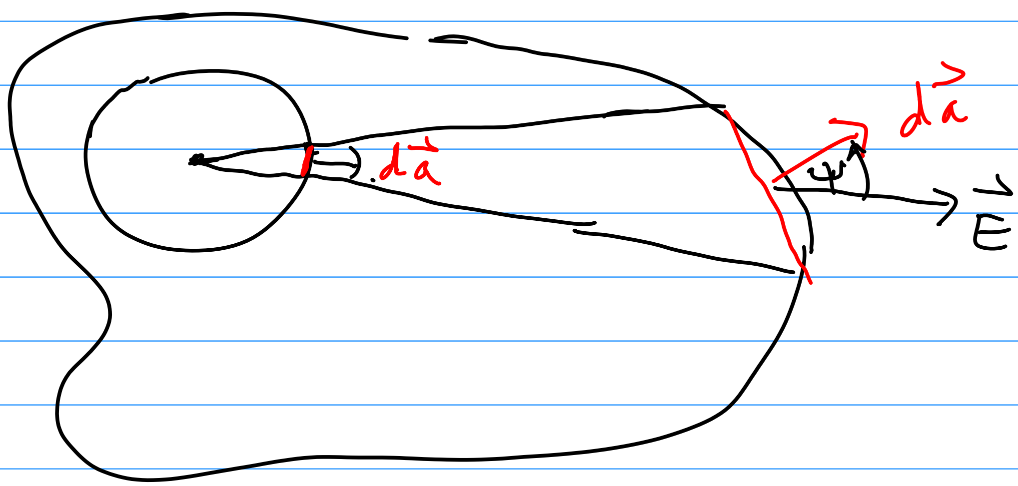
Review for the Monday 1/2 exam (30 minutes):

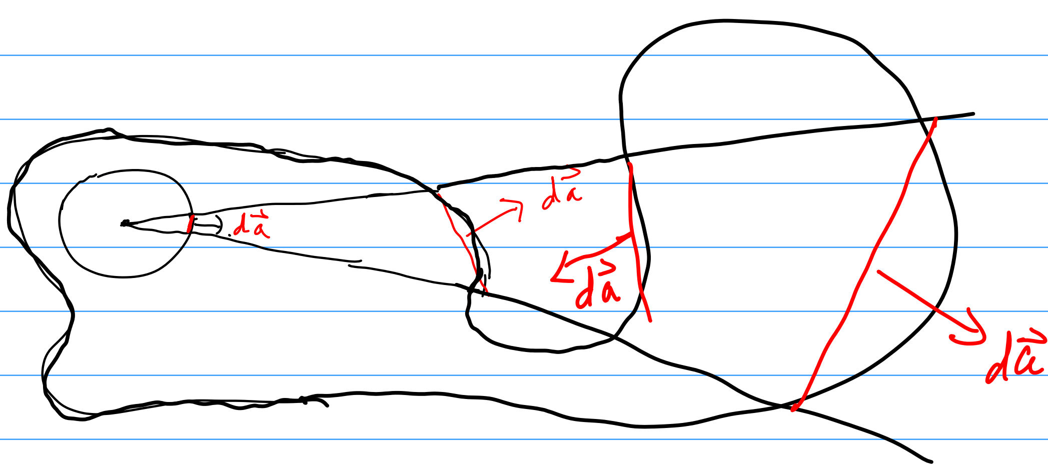
- -one problem on deriving areas or volumes of circles, cylinders, or spheres
- -two problems on setting up the integral for the electric field from either a line or surface charge distribution.

You will only have to justify the set-up of the integral with limits correctly in these problems.

InkSurvey muddy points:

- flux thru surfaces: $\int \vec{E}\cdot da \propto \int \frac{1}{r^2}\, da \cos\psi$





What happens with a charge offset from the center of the first sphere?

Solid angle: http://en.wikipedia.org/wiki/Solid_angle $\int d\Omega = \int \sin\theta\, d\theta\, d\phi = 4\pi$

$$- \oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\underbrace{\phantom{\oint \vec{E}\cdot d\vec{a}}}_{\text{neg flux}} \qquad \text{neg charge}$$

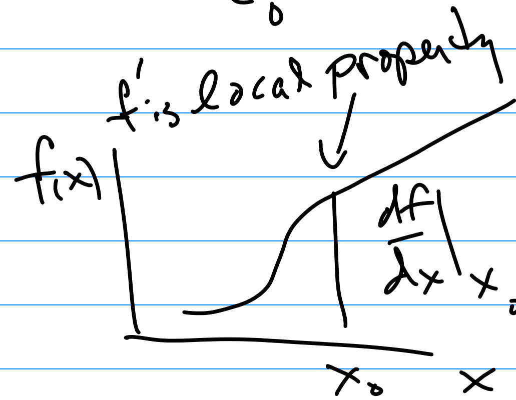
How do you get flux from differential form? $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$

$$\oint \vec{\nabla}\cdot\vec{E}\, d\tau = \frac{1}{\epsilon_0}\int \rho\, d\tau$$

$d\tau$: volume $dx\,dy\,dz$

div. theorem $\Downarrow$

$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$



Fundamental theorem of calculus

$$\int_a^b \frac{df}{dx}\, dx = f(b) - f(a)$$

sum changes in f to get total change at bndry

Integal of derivative over interval is the value of function at end points (bndries)

Fundamental theorem of diverences

$$\int \vec{\nabla}\cdot\vec{F}\, d\tau = \oint \vec{F}\cdot da$$

Integral of derivative (divergence) over region (volume) is value of function at bndry (surface)

- Analogy questions like $\vec{\nabla}\cdot\vec{B} = ?$ $\quad \vec{\nabla}\times\vec{E} = ?$

$$\oint \vec{B}\cdot d\vec{a} =$$

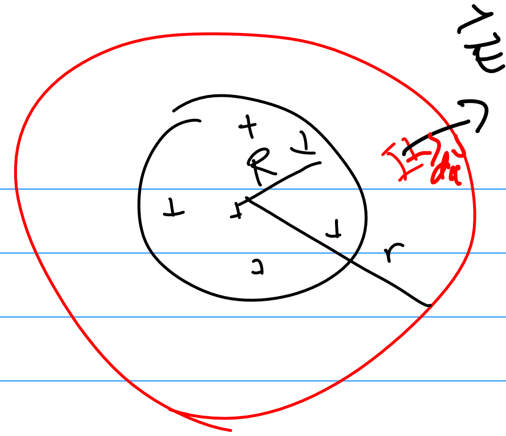
-Advantages of differential form: Coulombs law becomes a partial differential equation.

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$

Use numerical method to iterate solns

Examples of Gauss's law

uniformly charged sphere



$$\oint \vec E \cdot d\vec a = E\,4\pi r^2 \qquad E\,\cancel{4\pi} r^2 = \frac{\sigma\,\cancel{4\pi} R^2}{\epsilon_0}$$

$$\frac{Q_{encl}}{\epsilon_0} = \frac{\sigma\,4\pi R^2}{\epsilon_0} \qquad E = \frac{\sigma R^2}{\epsilon_0 r^2}$$

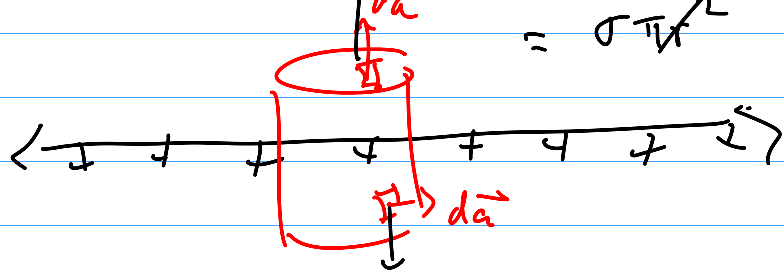
Close to the surface $r = R + \delta$

$$E = \frac{\sigma R^2}{\epsilon_0 (R+\delta)^2} = \frac{\sigma \cancel{R^2}}{\epsilon_0 \cancel{R^2}(1+\delta/R)^2}$$

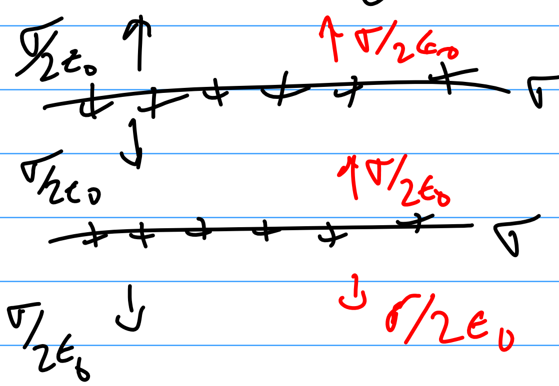
$$E = \frac{\sigma}{\epsilon_0}\left(1 - 2\frac{\delta}{R} + \dots\right)$$

Infinite plane of charge

$$\Phi = E\,\cancel{\pi r^2} + E\,\cancel{\pi r^2} = \sigma\,\cancel{\pi r^2}$$



Superposition principle



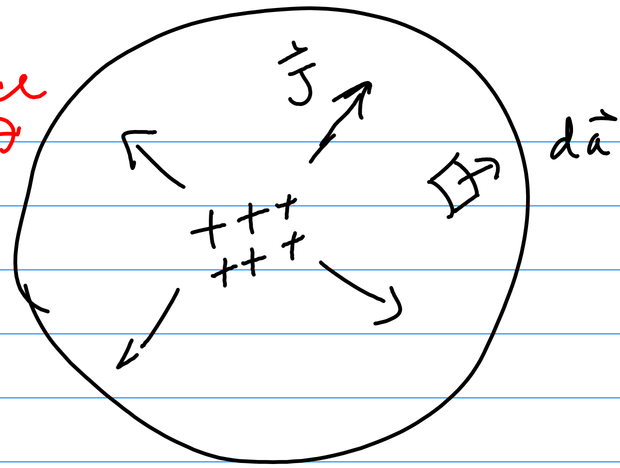
Applet: **Questions?**

http://www.cegep-ste-foy.qc.ca/freesite/fileadmin/users/29/NYB/Theorie/contactdiff.swf

Conservation of charge continued.

by this surface

$$\oint \vec{J} \cdot d\vec{a} = -\frac{dQ_{enclosed}}{dt}$$



$$Q_{enclosed} = \int \rho \, d\,volume$$

$$d\,volume = dx\,dy\,dz \text{ or } r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$-\frac{dQ_{enclosed}}{dt} = \int -\frac{\partial \rho}{\partial t} \, dx\,dy\,dz$$

$$\oint \vec{J} \cdot d\vec{a} = \int \vec{\nabla} \cdot \vec{J} \, dx\,dy\,dz \quad \text{Divergence theorem}$$

Differential form of conservation of charge is

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$

Also called the CONTINUITY EQUATION

http://en.wikipedia.org/wiki/Continuity_equation

this link gives examples for 4-current in special relativity, fluid dynamics, thermodynamics, quantum mechanics, general relativity, quantum chromodynamics

traffic flow continuity equation

Examples of vector calculus in fluids

http://en.wikipedia.org/wiki/Navier%E2%80%93Stokes_equations