Reading: Finish Chapter 11

Chapter 11: Radiation

Look at the fields from a source on a sphere very far away, $r \to \infty$: radiation zone.

$$P = \int_{\text{sphere}} \vec{S} \cdot d\vec{a} \qquad d\vec{a} = r^2 \sin\theta \, d\theta \, d\varphi \, \hat{r}$$

Find the dominant terms for $\vec{E}$ & $\vec{B}$ (typically $\sim \frac{1}{r}$)

so we can find $\vec{S}$ terms that will go like $\frac{1}{r^2} \hat{r}$

Problem 11.12

Ring is neutral

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau' \qquad \rho = 0$$

$$= 0$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

$$= \frac{\mu_0}{4\pi}\oint \frac{I(t-\mathscr{r}/c)}{\mathscr{r}}\,d\vec{l}\,'$$

$$\mathscr{r} = r\left[1 - \frac{b\sin\theta\cos\varphi'}{r}\right]$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 + \frac{b\sin\theta\cos\varphi'}{r}\right] \qquad t_0 = t - \frac{r}{c}$$

$$I(t-\mathscr{r}/c) = I(t_0) + \dot{I}(t_0)(t-\mathscr{r}/c - t_0)$$

$$= I(t_0) + \dot{I}(t_0)\left(t - \mathscr{r}/c - t + r/c\right)$$

$$= I(t_0) + \dot{I}(t_0)\frac{(r-\mathscr{r})}{c}$$

$$= I(t_0) + \dot{I}(t_0)\frac{b}{c}\sin\theta\cos\varphi'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\oint \frac{1}{r}\left[\underbrace{I(t_0)}_{\oint = 0} + \dot{I}(t_0)\frac{b}{c}\sin\theta\cos\varphi'\right]d\vec{l}\,'$$

$$= \frac{\mu_0 b^2}{4rc}\,\dot{I}(t_0)\sin\theta\,\hat{y}$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0 b^2}{4rc}\,\dot{I}(t_0)\sin\theta\,\hat{\varphi} \qquad \dot{I}(t_0) \to \dot{I}\left(t - \frac{r}{c}\right)$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} = -\frac{\mu_0 b^2}{4rc}\sin\theta\,\ddot{I}(t_0)\,\hat{\varphi}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$B_\theta = \frac{\mu_0 b^2}{4rc^2}\sin\theta\,\ddot{I}(t_0)$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{\mu_0}{16c^3\pi^2 r^2}\sin^2\theta\,\underbrace{\left(\pi\,\ddot{I}(t_0)\,b^2\right)}_{\ddot{m}(t_0)}{}^2\,\hat{r}$$

$$= \frac{\mu_0}{16\pi^2 c^3 r^2}\sin^2\theta\,\ddot{m}(t_0)^2\,\hat{r}$$

magnetic dipole moment

$$P_{TOT} = \int_{sphere}\vec{S}\cdot d\vec{a} = \boxed{\frac{\mu_0}{6c^3\pi}\ddot{m}^2 = P_{TOT}}$$