Note Title 2/28/2008

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0 = \frac{\rho_f}{\epsilon_0} + \frac{\rho_b}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{P} = -\rho_b$$

$$\vec{\nabla}\cdot\underbrace{\left(\epsilon_0\vec{E} + \vec{P}\right)}_{\vec{D}} = \rho_f$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f \Rightarrow \oint \vec{D}\cdot d\vec{a} = Q_{f_{\text{free}}\,\text{enclosed}}$$

Linear materials

$$\boxed{\vec{P} = \epsilon_0 \chi_e \vec{E}}$$

flux

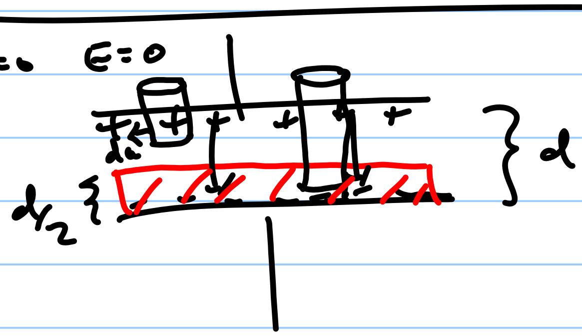
$$D\,4\pi r^2 = Q_f$$

$$D = \frac{Q_f}{4\pi r^2}$$

$$\epsilon E = \frac{Q_f}{4\pi r^2}$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon_0\vec{E} + \epsilon_0\chi_e\vec{E} = \underbrace{\epsilon_0(1+\chi_e)}_{\epsilon}\vec{E}$$

$$K = \frac{\epsilon}{\epsilon_0}$$



$$\oint \vec{D}\cdot d\vec{a} = Q_{f\text{en}}$$

$$D = \epsilon_0\vec{E} + \vec{P} = \epsilon_0\vec{E} + \epsilon_0\chi_e\vec{E}$$
$$= \epsilon\vec{E}$$

D_{vac} : $\Phi_D = D\cancel{A} = \sigma_f A$ $\quad$ $D_{vac} = \sigma_f = \epsilon_0 E + \cancel{P}^{\,0}_{vac}$

$$E_{vac} = \frac{\sigma_f}{\epsilon_0}$$

D_{rubber} : $\Phi_D = DA = \sigma_f A$ $\quad$ $D = \sigma_f$

$$\epsilon E = \sigma$$

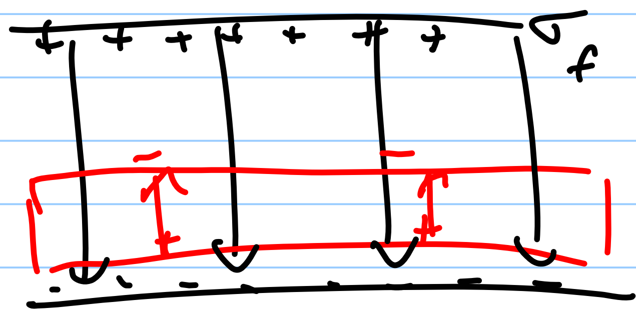
$$\boxed{E_{rubber} = \frac{\sigma_f}{\epsilon}}$$

$$K = \frac{\epsilon}{\epsilon_0}$$

$$P = \epsilon_0 \chi_e E = \epsilon_0 \chi_e \frac{\sigma}{\epsilon}$$

$$\sigma_{bound} = \vec{P}\cdot\hat{n} = \epsilon_0 \chi_e \frac{\sigma}{\epsilon}$$

$E_{vac} + E_{dipoles}$



σ_b

$$E_{rubber} = E_{\sigma_f} + E_{\sigma_b}$$

$$= \frac{\sigma_f}{\epsilon_0} - \frac{\sigma_b}{\epsilon_0}$$

$$D = \epsilon_0 E + P = \sigma_f$$

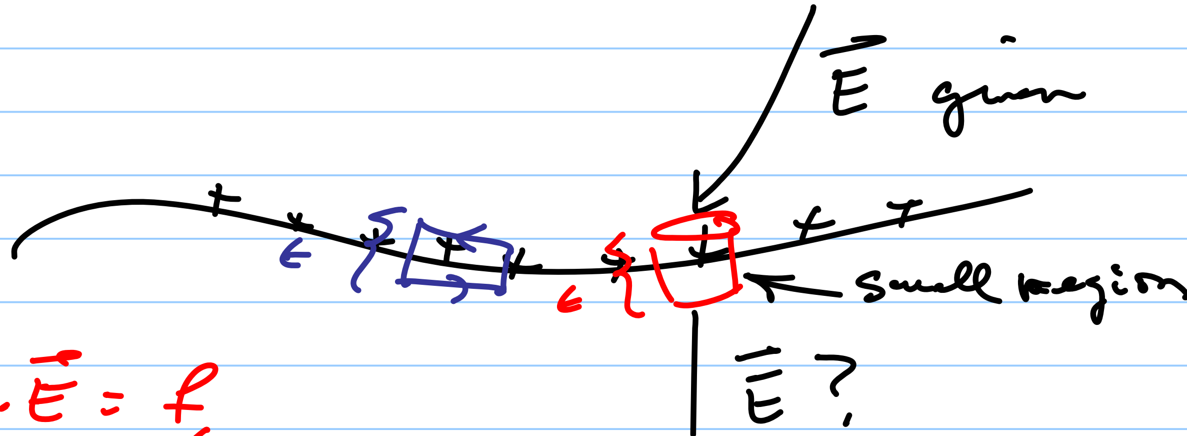
field due to bound charg

$$E = \frac{\sigma_f}{\epsilon_0} - \frac{P}{\epsilon_0} = \frac{\sigma_f}{\epsilon_0} - \frac{\epsilon_0 \chi_e \, \sigma_f/\epsilon}{\epsilon_0}$$

field from charge with dielectr.

$$E_{rubber} = \frac{\sigma_f}{\epsilon}$$

bndy conditions



let $\epsilon \to 0$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

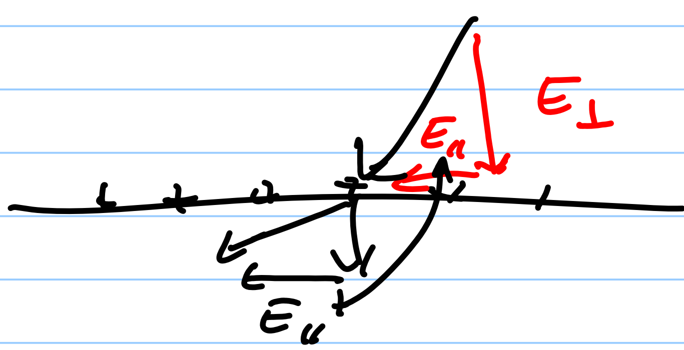
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$$

$$\left(\vec{E} \cdot \Delta\vec{a}\right)_{above} - \left(\vec{E} \cdot \Delta\vec{a}\right)_{below} = \frac{\sigma\, da}{\epsilon_0}$$

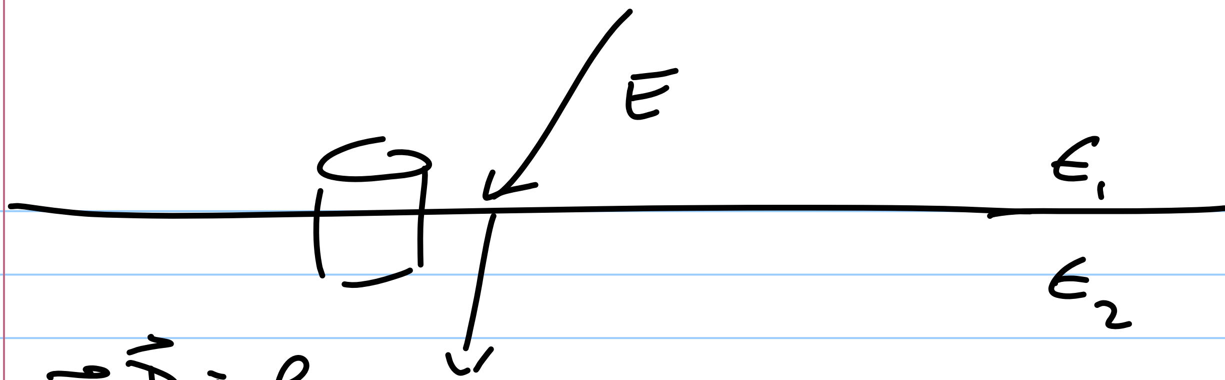
$$\boxed{E_\perp^{above} - E_\perp^{below} = \frac{\sigma}{\epsilon_0}}$$

$$\vec{\nabla} \times \vec{E} = 0 \qquad \oint \vec{E} \cdot d\vec{l} = \int \underbrace{\vec{\nabla} \times \vec{E} \cdot d\vec{a}}_{0} \quad da \to 0$$

$$E_\parallel^{above} - E_\parallel^{below} = 0$$



$$E_\perp^{above} - E_\perp^{below} = \frac{\sigma}{\epsilon_0}$$



$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\oint \vec{D}\cdot d\vec{a} = Q_f$$

$$D_\perp^{above} - D_\perp^{below} = \sigma_f$$

$$\epsilon_1 E_\perp^{above} - \epsilon_2 E_\perp^{below} = \sigma_f$$

$$E_\parallel^{above} - E_\parallel^{below} = 0$$

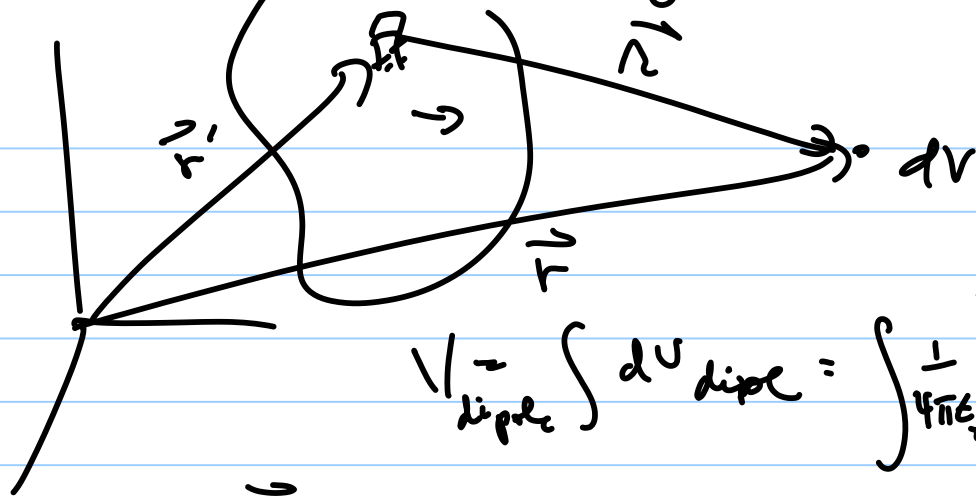
$\boxed{\text{Review}}$

$\rightarrow$ dipole mom $\qquad \vec{p} = \sum_i q_i \vec{r}_i$

$$\vec{p} = \int dq\, \vec{r} = \int \rho\, d\tau\, \vec{r}$$

$$\_\,\hat{x} + \_\,\hat{y} + \_\,\hat{z}$$

→ know how we started our treatment of fields in matter



$$V_{dipole} = \int dV_{dipole} = \int \frac{1}{4\pi\epsilon_0} \frac{\vec{P}\, d\tau' \cdot \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$d\vec{p} = \vec{P}\, d\tau$$

↑ dipole mom/vol

This resulted in $V = \int \frac{1}{4\pi\epsilon_0} \frac{\rho}{\mathscr{r}} d\tau'$

$$\frac{1}{4\pi\epsilon_0}\left[\oint \frac{\sigma_b\, da'}{\mathscr{r}} + \int \frac{\rho_b\, d\tau'}{\mathscr{r}}\right]$$

where $\sigma_b = \vec{P}\cdot\hat{n}$; $\rho_b = -\vec{\nabla}\vec{P}$

→ $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$; $\vec{\nabla}\cdot\vec{D} = \rho_{free}$

OR

$$\oint \vec{D}\cdot d\vec{a} = Q_{free}$$

⇒ LINEAR material

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

→ know how to calculate $\vec{D}$ as in lecture examples

→ from this $\vec{D}$ be able to calculate $\vec{E}$ everywhere

→ from this $\vec{E}$ be able to calculate $\vec{P}$ for a linear material

→ from this $\vec{P}$ be able to calculate σ_b & ρ_b

→ Boundary conditions (see above)

$$\epsilon_1 E_\perp^{\text{above}} - \epsilon_2 E_\perp^{\text{below}} = \sigma_f$$

$$E_\parallel^{\text{above}} - E^{\text{below}} = 0$$