Antenna performance.

Hertz dipole antenna: $(d \ll \lambda)$

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{2}\frac{\pi}{c}\left(\frac{d}{\lambda}\right)^2 I_0^2 \sin^2\theta$$

$$\langle P \rangle = \frac{1}{2}\underbrace{\frac{8\pi^2}{3c}\left(\frac{d}{\lambda}\right)^2}_{\text{rad. resistance.}} I_0^2$$

center-fed linear antenna:

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{2}\frac{1}{\pi c}\left(\frac{\cos\left(\frac{kd}{2}\cos\theta\right) - \cos\left(\frac{kd}{2}\right)}{\sin\theta}\right)^2 I_0^2$$

this gives the angular distrib. of power.

$$\langle P \rangle_{tot} = \int_{4\pi} \left\langle \frac{dP}{d\Omega} \right\rangle d\Omega$$

how much of the power is directed?

gain $G \equiv \dfrac{\langle dP/d\Omega \rangle_{max}}{P_{tot}/4\pi}$

if antenna is isotropic - gain = 1

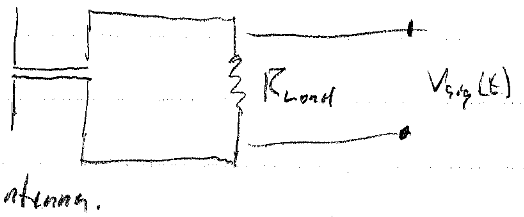
Hertz dipole: $G = 3/2 = 1.5$

$\lambda/2$ linear $G = 1.64$

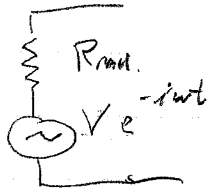
High gain possible for $l \gg \lambda$ (or use an array)

receivers

EM waves that are broadcast induce currents in rec. antennas.



antenna is like a current source →



Ideally, match R_{rad} to R_{load}.

in this case → maximum power transfer:

$$P_{max} = \frac{\mathcal{E}_0^2}{8R_{rad}} \quad \text{(see HW)} \qquad \mathcal{E}_0 = \text{ind. emf (voltage)}$$

input wave has an intensity $\langle S \rangle = \frac{\text{power}}{\text{area}}$

→ recieved power is $P_{max} = \langle S \rangle A$

↳ effective area.

$$\mathcal{E}_0 = (E_{rad})_{peak}\, l = E_{in}\, l$$

$$\langle S \rangle = \frac{c}{8\pi} E_{in}^2$$

$$\langle P_{max} \rangle = \frac{(E_{in}\, l)^2}{8R_{rad}} = \frac{c}{8\pi} E_{in}^2 A$$

$$\to A_{Hertz} = \frac{\pi l^2}{cR_{rad}} = \frac{\pi l^2}{c} \frac{3c}{8\pi^2}\left(\frac{\lambda}{l}\right)^2 = \frac{3}{8\pi}\lambda^2$$

general antenna:

$$A \doteq G A_0 \qquad A_0 = (\lambda/2\pi)^2$$

communications link.

$$P_{out} = G_r \cdot A_0 \cdot S$$

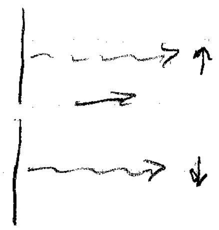
received

$$= G_r \cdot A_0 \cdot \frac{G_t \cdot P_{in}}{4\pi r^2}$$

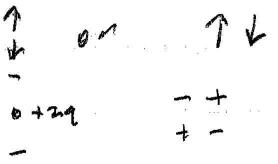
$$\frac{P_{out}}{P_{in}} = G_r G_t \cdot \frac{\lambda^2}{4\pi} \cdot \frac{1}{4\pi r^2} = \frac{A_r A_t}{\lambda^2 r^2}$$

Can improve gain by making antenna arrays.

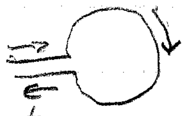
- a small dipole is like a $\lambda/2$ antenna.
- simulate λ antenna by two small dipoles separated by $d/2$



can also get radiation from quadrupoles:



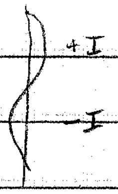
or magnetic dipoles



or any other multipole moment.

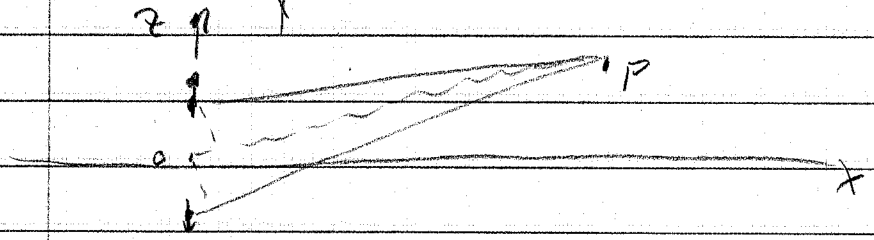
Antenna arrays: qualitative picture

Full wave linear antenna



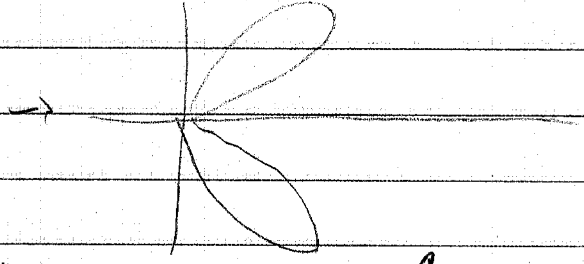
dipole pair model

180° out of phase.



Add E-fields from each dipole
- account for phase delay

where is field → 0?
- on x-axis b/c dipoles are out of phase.

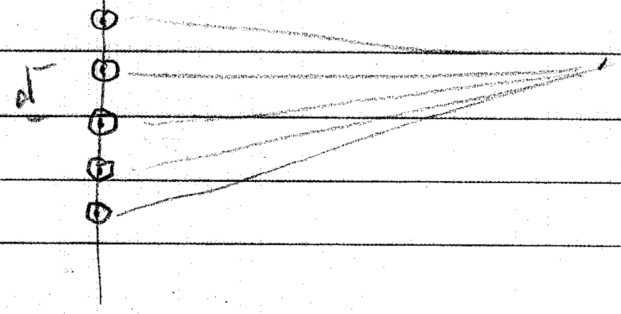


axial quadrupole ↑

lateral ↓

↑ ↓

Extension to arrays:



each dipole has ampl. + phase.

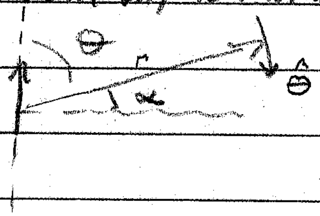
$m\lambda = d\sin\theta$

For uniform phase.

Antenna arrays - dipole modeling

- powerful simulation based on simple ideas

individual dipole radiator:



$$\vec{E}_\theta = \frac{[\ddot{p}]}{c^2 r} \sin\theta \, \hat{\theta}$$

note polarization is in same orientation as dipole

- linearly polarized, no $\hat{\phi}$, $\vec{E} \perp \hat{r}$

field is maximum at $\theta = \pi/2$

> change angle to $\alpha = \pi/2 - \theta$

$$E_\alpha = \frac{[\ddot{p}]}{c^2 r} \cos\alpha$$

$$p(t) = p_0 e^{-i\omega t} \rightarrow \ddot{p} = -\omega^2 p_0 e^{-i\omega t}$$

$$[\ddot{p}] = -\omega^2 p_0 e^{-i\omega(t - r/c)}$$

where we are evaluating at the retarded time.

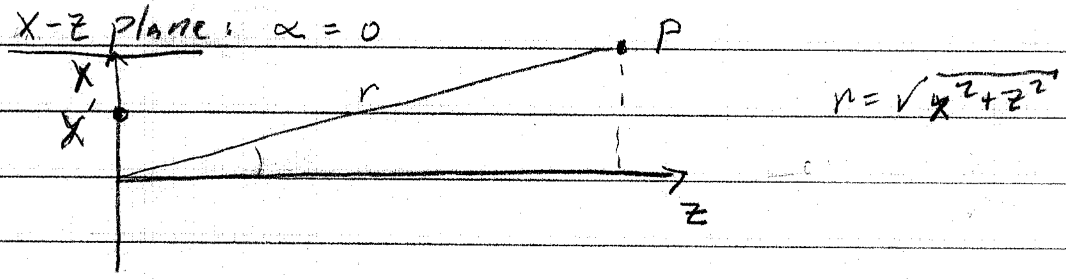
$$E_\alpha = -\frac{\omega^2}{c^2 r} \cos\alpha \, e^{i(\frac{\omega}{c} r - \omega t)}$$

$\omega/c = k$ in vacuum.

$$E_\alpha = -\frac{k^2}{r} \cos\alpha \, e^{i(kr - \omega t)}$$

we'll evaluate at $t = 0$, drop sign.

$\vec{p} = p_0 \hat{y}$



$$E_y = \frac{p_0 k^2}{r} e^{ikr} \quad \text{with } r \to \sqrt{x^2+z^2}$$

add shift: $x \to x - x'$

this is form of spherical wave.

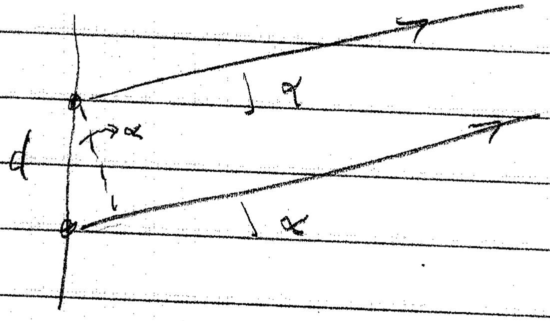
y-z plane: $\cos\alpha = z/r$

$$E_y = E_0 \cos\alpha \quad \to \quad \frac{p_0 k^2 \cos^2\alpha}{r} e^{ikr}$$

$$E_z = E_0 \sin\alpha \quad \to \quad \frac{p_0 k^2 \cos\alpha \sin\alpha}{r} e^{ikr}$$

Far-field pattern: Bragg law

At far field, spherical waves are close to plane waves.



optical path difference is $d \sin\alpha$

Far field is sum:

$$E \sim \frac{1}{r} \sum_j e^{ik(r + j d \sin\alpha)}$$

→ max when $kd \sin\alpha = 2\pi m$

$\to m\lambda = d \sin\alpha$

Bragg law.

Note that constant phase shift among sources → angular shift in pattern.

examples: phased array radar

VLA telescope