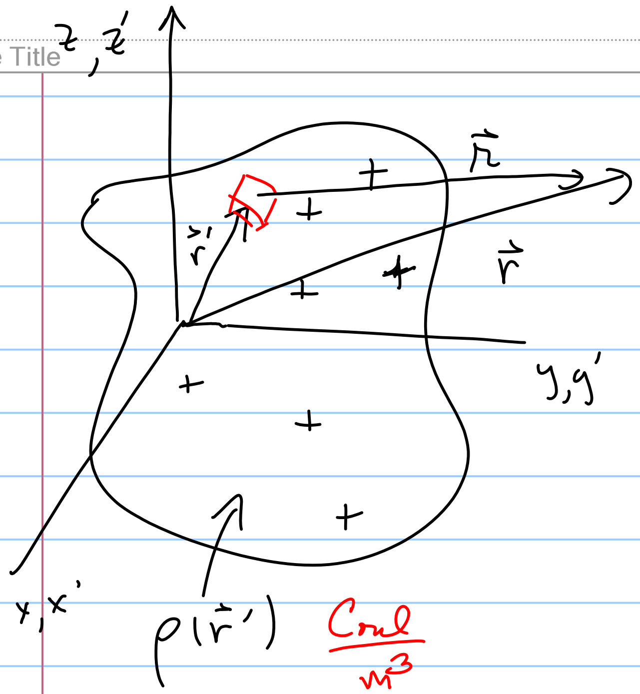
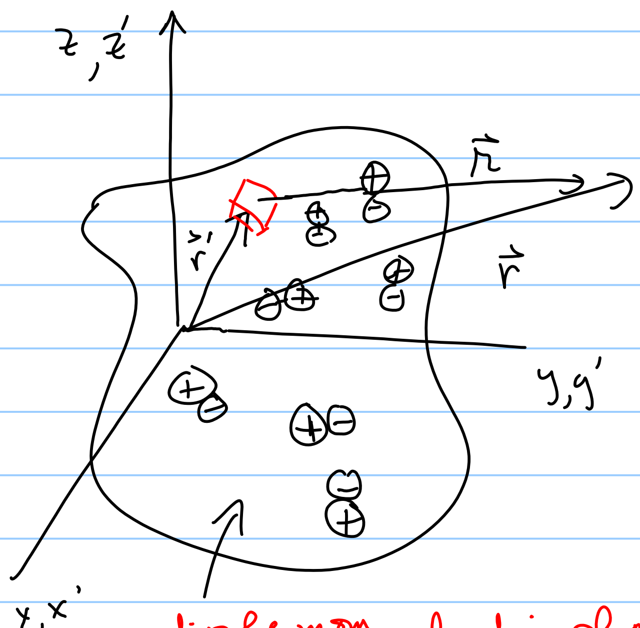
Note Title 2/19/2009



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')\,d\tau'}{\mathscr{r}} + \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\,d\tau'}{\mathscr{r}}$$

$$V_{\text{pt charge}} = \frac{1}{4\pi\epsilon_0}\frac{q_i}{\mathscr{r}}$$



dipole mom / m³ glued in place

$$V(\vec{r}) = \,?$$

$$V_{\text{one dipole}} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$V(\vec{r}) = \sum_i \frac{1}{4\pi\epsilon_0}\frac{\vec{p}_i\cdot\vec{\mathscr{r}}}{\mathscr{r}^3}$$

Dipole is NOT at origin but at $\vec{r}'$

For a continuous distribution of charge $q_i \to dq = \rho\, d\tau$ (charge/vol)

For a continuous distribution of dipoles $\vec{p}_i \to d\vec{p} = \vec{P}\, d\tau$

$\vec{P}$ is $\frac{\text{dipole moment}}{\text{vol}}$

$$V(\vec{r}) = \sum_i \frac{1}{4\pi\epsilon_0} \frac{\vec{P}_i \cdot \vec{\mathscr{r}}}{\mathscr{r}^3} \longrightarrow \int \frac{1}{4\pi\epsilon_0} \frac{\vec{P}\, d\tau \cdot \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\vec{P} \cdot (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d\tau'$$

Note $\quad \vec{\nabla}' \dfrac{1}{|\vec{r} - \vec{r}\,'|} = \dfrac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3} \quad \& \quad \vec{\nabla} \dfrac{1}{|\vec{r} - \vec{r}\,'|} = -\dfrac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$

$$\vec{\nabla}' \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} = \hat{x} \frac{\partial}{\partial x'} \frac{1}{\sqrt{}} + \hat{y} \frac{\partial}{\partial y'} \frac{1}{\sqrt{}} + \hat{z} \frac{\partial}{\partial z} \frac{1}{\sqrt{}}$$

$$\vec{\nabla} \cdot f\vec{A} = f\vec{\nabla} \cdot \vec{A} + \vec{A} \cdot \vec{\nabla} f$$

$$\vec{A} \cdot \vec{\nabla} f = \vec{\nabla} \cdot f\vec{A} - f\vec{\nabla} \cdot \vec{A}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \vec{P} \cdot \vec{\nabla}' \left(\frac{1}{|\vec{r} - \vec{r}\,'|} \right) d\tau'$$

$$\vec{A} \cdot \vec{\nabla} (f) = \vec{\nabla} \cdot f\vec{A} - f\, \vec{\nabla} \cdot \vec{A}$$

$$V = \frac{1}{4\pi\epsilon_0} \left[\int \vec{\nabla}' \cdot \frac{\vec{P}\, d\tau'}{|\vec{r} - \vec{r}\,'|} - \int \frac{1}{|\vec{r} - \vec{r}\,'|} \vec{\nabla}' \cdot \vec{P}\, d\tau' \right]$$

DIVERGENCE THEOREM

$$= \frac{1}{4\pi\epsilon_0} \oint \frac{\vec{P} \cdot d\vec{a}\,'}{|\vec{r} - \vec{r}\,'|} + \frac{1}{4\pi\epsilon_0} \int \frac{-\vec{\nabla} \cdot \vec{P}}{|\vec{r} - \vec{r}\,'|} d\tau'$$

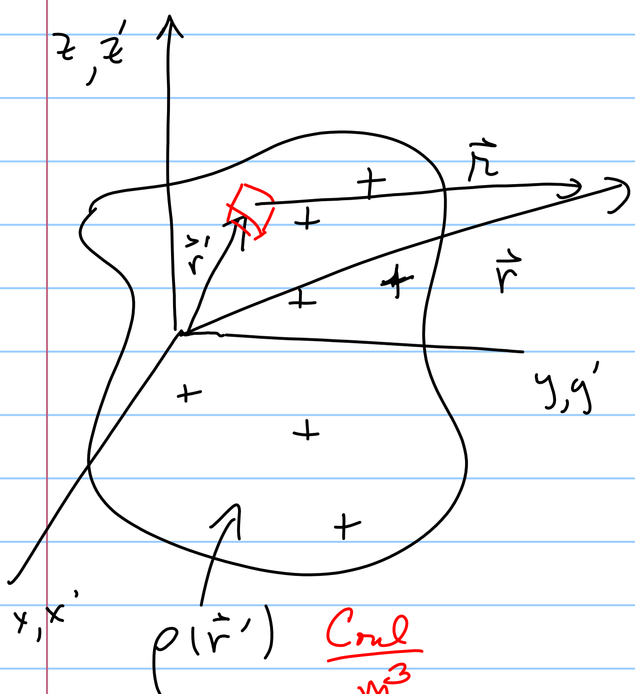
over surface of material

$d\vec{a}\,' = \hat{n}\, da$ where $\hat{n}$ is $\perp$ to $d\vec{a}$

$$= \frac{1}{4\pi\epsilon_0} \oint \frac{\sigma_b \, da}{|\vec{r} - \vec{r}\,'|} + \frac{1}{4\pi\epsilon_0} \int \frac{\rho_b \, d\tau'}{|\vec{r} - \vec{r}\,'|}$$

where $\sigma_b = \vec{P} \cdot \hat{n}$ $\qquad \rho_b = -\vec{\nabla} \cdot \vec{P}$

Compare with the result at the beginning of this lecture



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,') d\tau'}{\mathscr{r}} + \frac{1}{4\pi\epsilon_0} \int \frac{\sigma \, d\tau'}{\mathscr{r}}$$

$$V_{\text{pt charge}} = \frac{1}{4\pi\epsilon_0} \frac{q_i}{\mathscr{r}}$$