

ROW REDUCTION

Text: 7.3.289-7.3.294

Lecture Notes: N/A

Lecture Slides: N/A

Quote of Short Homework One

Durante degli Alighieri: Consider your origin; you were not born to live like brutes, but to follow virtue and knowledge.

Canto XXVI : The Inferno (1308-1321)

1. GOALS

The goal of this assignment is to expose the student to *Gauss elimination* or the so-called row reduction algorithm.¹ This algorithm is the most important of linear algebra and is REQUIRED for all future study. After this assignment the student should:

- Know the three elementary row-operations that can be applied to a matrix.
- Understand how to algorithmically apply these row-operations to a matrix in order to derive the reduced row-echelon form of the matrix.

2. OBJECTIVES

To achieve the previous goals the student will meet the following objectives:

- (1) Attend and take notes during the first lecture and ask any introductory questions they might have after the row-reduction example.
- (2) Read section 7.3 of the text book paying particular attention to pages 289 through 294.² Specifically, if the students fail to follow the in-class example they should work through the steps example 3 and example 4.³
- (3) Use GE on a set of matrices and from this set find each of their reduced row-echelon forms.
- (4) Check the GE work using a computational device and correct any errors found.

3. PROBLEMS

Using Gaussian elimination, find the *reduced row-echelon form* for each of the following matrices. You may find it useful to check your work with some sort of computational device you are comfortable with. If you do not have a preference then I offer the following website, which also highlights the steps they use: <http://www.math.ou.edu/~bogacki/cgi-bin/lat.cgi?c=rref>⁴ For \mathbf{A}_6 and \mathbf{A}_7 , h is a parameter. You can check your work by choosing a value for h at the final step and comparing to a computational device.

$$\mathbf{A}_1 = \begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 6 & 18 & -4 & 20 \\ -1 & -3 & 8 & 4 \\ 5 & 15 & -9 & 11 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 3 & 0 \end{bmatrix}, \mathbf{A}_4 = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 2 & 4 & 6 & 20 \\ 3 & 6 & 9 & 30 \end{bmatrix}, \mathbf{A}_5 = \begin{bmatrix} 5 & 3 & 22 \\ -4 & 7 & 20 \\ 9 & -2 & 15 \end{bmatrix},$$

$$\mathbf{A}_6 = \begin{bmatrix} 1 & -5 & 1 \\ -1 & 7 & 1 \\ -3 & 8 & h \end{bmatrix}, \mathbf{A}_7 = \begin{bmatrix} 1 & -3 & 5 \\ -3 & 9 & -7 \\ 2 & -6 & h \end{bmatrix}, \mathbf{A}_8 = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{bmatrix}, \mathbf{A}_9 = \begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix}, \mathbf{A}_{10} = \begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix},$$

$$\mathbf{A}_{11} = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}.$$

¹Words written in italics are often hyperlinks. On a computer you should be able to click on them and go to a webpage.

²To tell someone to read a passage is a rather broad directive. For instance, I am particularly bad at 'skimming.' I have been reading textbooks for so long that I tend to read word-for-word. This is a detriment. When I am asking you to read something out of the text you must decide for yourself how to go about this task. If you are not an efficient reader then this process will take time, you might as well start practicing now.

³It is expected that at this point much of the language and motivation of these examples is, at this point, undefined. The goal here is reproducing the algorithm. The language takes time and this is only the first day.

⁴They may use different steps than you but if all goes well then the reduced row-echelon forms should be the same. That is to say, every matrix has a unique reduced row-echelon form.