MATH348 - November 16, 2009 **NAME**:

Exam II - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) True/False and Short Response

   (a) Mark each statement as either true or false. No justification is needed.

      i. Real Fourier series are used to represent periodic functions that complex Fourier series cannot.

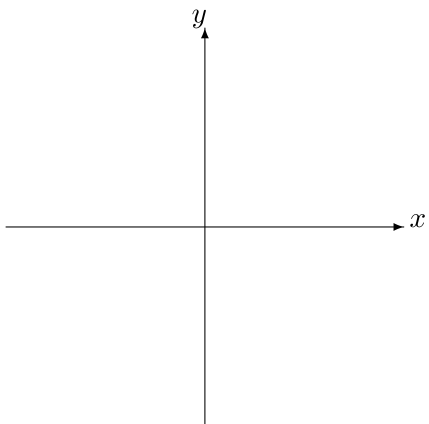
      ii. Every function that is not periodic can have a periodic extension.

      iii. If a periodic function is neither odd nor even then the Fourier series representation will have both sine and cosine functions.

      iv. Suppose f is a periodic function such that $f(-x) = f(x)$. The Fourier series representation of f will have only sine functions.

   (b) How is the Fourier integral related to the Fourier series? What is the purpose of each?

   (c) Given the graph for the function f:



   Does f have a Fourier series representation? If so then will the series contain cosine functions?

2. (10 Points) Given the following integrals,

$$\int_{-\pi}^{\pi} f(x)dx = 0,$$

$$\int_{-\pi}^{\pi} f(x)\cos(nx)dx = \left[\frac{\sin(nx)}{n}\bigg|_{-\pi}^{0} + \frac{\sin(nx)}{n}\bigg|_{0}^{\pi} \right],$$

$$\int_{-\pi}^{\pi} f(x)\sin(nx)dx = \left[\frac{\cos(nx)}{n}\bigg|_{-\pi}^{0} - \frac{\cos(nx)}{n}\bigg|_{0}^{\pi} \right],$$

$$\int_{-\pi}^{\pi} g(x)dx = e^{-in\pi} - e^{in\pi},$$

$$\int_{-\pi}^{\pi} g(x)e^{-inx}dx = \frac{i}{n}\left[e^{-inx}\big|_{0}^{\pi} - e^{inx}\big|_{-\pi}^{0} \right],$$

$$\frac{e^{i\omega} - e^{-i\omega}}{2i\omega} = \int_{-\infty}^{\infty} h(x)e^{-i\omega x}dx,$$

where n is an integer and $\omega \in \mathbb{R}$.

(a) Calculate the real Fourier series of $f(x)$.

(b) Calculate the complex Fourier series of $g(x)$.

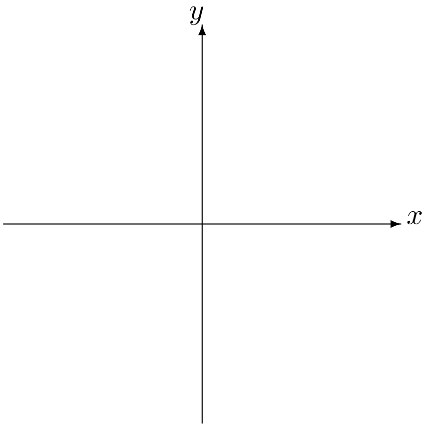
(c) Calculate the Fourier transform of $h(x)$.

(d) Determine the symmetry of the function $f(x)$.

(e) Calculate the real Fourier series representation of $g(x)$.

3. (10 Points) Let $f(x) = 1,\ x \in (-1, 1),\ f(x + 2) = f(x)$. Find the COMPLEX Fourier series representation of f.

4. (10 Points) Below is the graph of the function f.



(a) Graph the Fourier cosine half-range expansion of f with a dashed line and the Fourier sine half-range expansion with a solid line on the axes below.

(b) Find the Fourier sine series half-range expansion of f.

5. (10 Points)

(a) Let $\hat{f}(\omega) = \delta(\omega + 1) - \delta(\omega - 1)$. Find the inverse Fourier transform of $\hat{f}$.

(b) Suppose that f is given as,

$$f(x) = \begin{cases} -x^2 + 1, & -1 \le x \le 1 \\ 0, & \text{otherwise} \end{cases} . \tag{1}$$

Calculate the complex Fourier transform of f.