MATH 348 - Advanced Engineering Mathematics April 23, 2008
Homework 10, Spring 2008 **Due**: April 30, 2008

LINEAR ALGEBRA - VECTOR SPACES, DETERMINANTS, INVERSE MATRICES AND EIGENVALUE PROBLEMS

1. Given,

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}.$$

   (a) What is the basis and dimension of Nul $\mathbf{A}$?
   (b) What is the basis and dimension of Col $\mathbf{A}$?
   (c) What is the basis and dimension of Row $\mathbf{A}$?
   (d) What is the Rank of $\mathbf{A}$?

2. Given,

$$\mathbf{A} = \begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix}.$$

   Determine $\mathbf{A}^{-1}$ via:

   (a) Calculate $\det(\mathbf{A})$.
   (b) The Gauss-Jordan Method (pg.317).
   (c) The cofactor representation (Theorem 2 pg.318).
   (d) Check your result by showing $\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$

3. Given,

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$

   (a) Determine the eigenvalues of $\mathbf{A}$.
   (b) Determine the eigenvectors of $\mathbf{A}$.

   Hint: Use a cofactor expansion along the first row to get $(4-\lambda)(1-\lambda)^2 + 2(1-\lambda) = 0$.

4. Given,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}. \tag{1}$$

   Calculate $\mathbf{A}^{1000}$ and $\mathbf{A}^{1001}$.

5. Let,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{2}$$

   Show that the eigenvectors of $\mathbf{A}$ are orthonormal.