1_14_08

Note Title 1/11/2008

A random variable is a function that assigns a numerical value to all possible outcomes of a random experiment.

i.e a random variable is a function that maps events to numbers

Example coin tossing. A coin is tossed 3 times and the outcome is noted.

Sample space = all possible outcomes

$$\Omega = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

Let the R.V. NH be the number of heads

outcome	HHH	HHT	HTH	THH	HTT	THT	TTH	TTT
NH	3	2	2	2	1	1	1	0

$P[NH=0] = 1/8$ $\quad P[NH=2] = 3/8$

$P[NH=1] = 3/8$ $\quad P[NH=3] = 1/8$

$$\langle NH \rangle = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8}$$

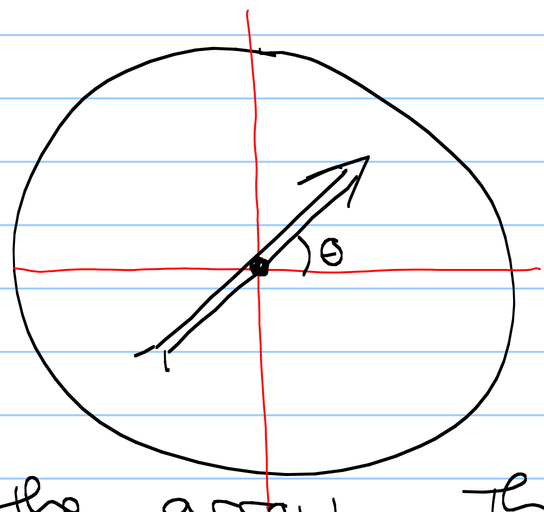
$$= \frac{12}{8} = \frac{3}{2}$$

in any given trial [i.e coin tossing] you cannot have $3/2$ heads, but in a long series of trials this would be the average value.

Parzen's Theory of Probability

> A random (or chance) phenomenon is an empirical phenomenon characterized by the property that its observation under a given set of circumstances does not always lead to the same observed outcomes (so that there is no deterministic regularity) but rather to different outcomes in such a way that there is statistical regularity. By this is meant that numbers exist between 0 and 1 that represent the relative frequency with which the different possible outcomes may be observed in a series of observations of independent occurrences of the phenomenon. ... A random event is one whose relative frequency of occurrence, in a very long sequence of observations of randomly selected situations in which the event may occur, approaches a stable limit value as the number of observations is increased to infinity; the limit value of the relative frequency is called the probability of the random event.

The sample space for coin tossing is discrete:



spin the arrow. The outcome can be any angle between $[0, 2\pi]$.

This is a continuous random phenomenon.

Assume spinner is horizontal. Then we believe that if the spinner is fair, any angle is equally likely.

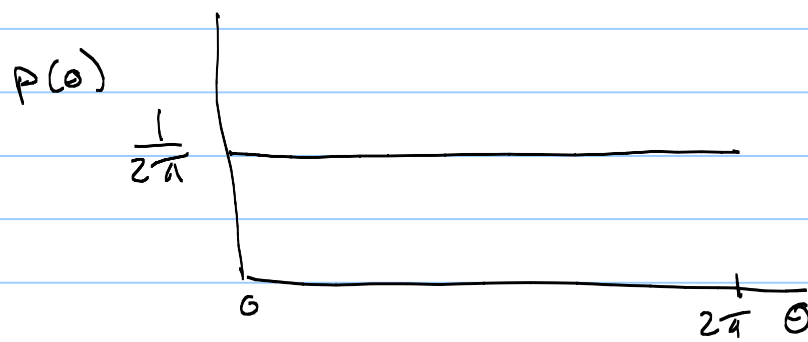
Note that it makes no real sense to ask the probability that the angle will be exactly equal to some particular angle.

What is the probability that θ will be exactly, say, π? 0. However

$$P\left[\pi/2 \le \theta \le \pi\right] = \frac{\pi/2}{2\pi} = \frac{1}{4}$$

$$p(\theta)\,d\theta = \frac{d\theta}{2\pi}$$

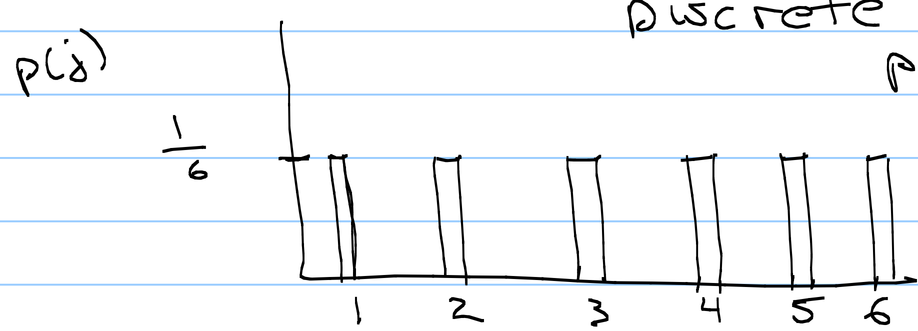
We say the probability is uniform.



$$\int_0^{2\pi} p(\theta)\,d\theta = 1$$

Die tossing

Discrete uniform probability



$$\text{mean}[\theta] = \int_0^{2\pi} \theta \, p(\theta) \, d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \theta \, d\theta = \pi \equiv \bar{\theta}$$

we also call this the Expectation of θ W.R.T. $p(\theta)$

$$\text{variance} \equiv E[(\theta - \bar{\theta})^2]$$

$$= E(\theta^2) - \bar{\theta}^2$$

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for the spinner

$$\text{variance}(\theta) = \frac{\pi^2}{3}$$

in this course we will write the expectation with $\langle \; \rangle$

E.g.

$$\bar{x} = E[x] = \langle x \rangle = \int_{-\infty}^{\infty} x p(x) \, dx$$

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x)\, p(x)\, dx$$

$$\text{variance} \equiv \sigma^2 = \langle (x - \bar{x})^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

Gaussian Mathematica
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$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-(x-m)^2/2\sigma^2}$$

find $x_0 \ni f(x_0) = \frac{1}{2} f(x_{max})$

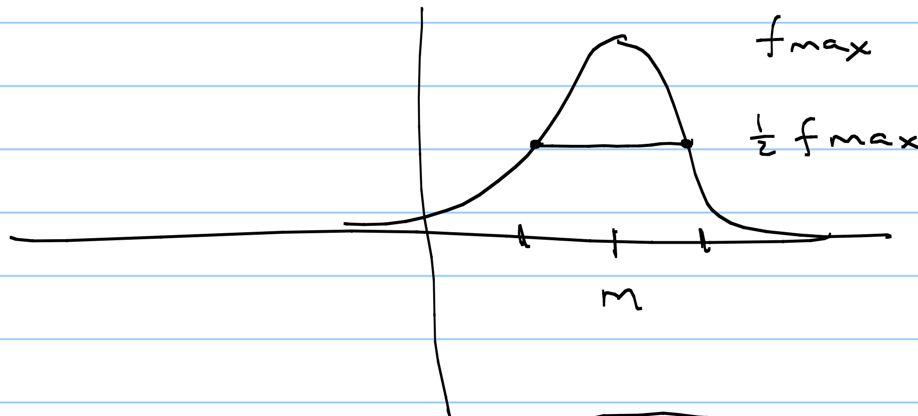
i.e. $$e^{-(x_0-m)^2/2\sigma^2} = \frac{1}{2} e^0 = \frac{1}{2}$$

$$-(x_0-m)^2/2\sigma^2 = -\ln 2$$

$$(x_0 - m)^2 = 2\sigma^2 \ln 2$$

$$x_0 - m = \pm \sigma \sqrt{2 \ln 2}$$

$$x_0 = m \pm \sigma \sqrt{2 \ln 2}$$



so $$\text{FWHM} = \underbrace{2\sqrt{2\ln 2}}_{\sim 2.3}\,\sigma$$

$\sim 2.3\,\sigma$