Homework 4:

1) a) Show using Maxwell's Eqns and the definitions from Chilwell's paper of u, v, w that

$$v = \frac{\gamma}{ik\alpha}\frac{du}{dx} \; ; \; w = \frac{\beta\gamma}{\alpha}u \; ; \; u = \frac{1}{ik\gamma\alpha}\frac{dv}{dx}$$

for <u>both</u> TE and TM polarization.

b) Using the boundary conditions we already know for $\vec{E}$ and $\vec{B}$, solve for boundary conditions at an interface for u, v, w. Once again, do this for <u>both</u> polarizations.

c) For a right going wave (positive k_x), show

$$v^+ = \gamma u^+ \quad \text{(or)} \quad \begin{pmatrix} u^+ \\ v^+ \end{pmatrix} = \begin{pmatrix} 1 \\ \gamma \end{pmatrix} u^+$$

Show for the left going wave

$$\begin{pmatrix} u^- \\ v^- \end{pmatrix} = \begin{pmatrix} 1 \\ -\gamma \end{pmatrix} u^-$$

d) Use the fact that

(1) u can be written a $u = u^+ + u^-$ (or) $\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ \gamma \end{pmatrix} u^+ + \begin{pmatrix} 1 \\ -\gamma \end{pmatrix} u^-$

$$\left.\begin{aligned} &(2)\; u^+(x_{j-1}) = u^+(x_j)\, e^{-ik\alpha_j(x_j - x_{j-1})} \\ &u^-(x_{j-1}) = u^-(x_j)\, e^{+ik\alpha_j(x_j - x_{j-1})} \end{aligned}\right\} \text{ \underline{Explain} why these are true.}$$

[Define $\Phi_j = k\alpha_j(x_j - x_{j-1})$]

$$(3)\; e^{\pm i\underbrace{k\alpha_j(x_j - x_{j-1})}_{\Phi_j}} = \cos(\Phi_j) \pm i\sin(\Phi_j)$$

to show that $\begin{pmatrix} u_{j-1} \\ v_{j-1} \end{pmatrix} = M_j \begin{pmatrix} u_j \\ v_j \end{pmatrix}$ where

$$M_j = \begin{pmatrix} \cos\Phi_j & -\frac{i}{\gamma_j}\sin\Phi_j \\ -i\gamma_j\sin\Phi_j & \cos\Phi_j \end{pmatrix}$$

2) a) Write down r_{cs} and t_{cs} from the paper.

b) Write down R and T from the paper and explain what they are.

c) Write down the time averaged power flows from the paper.

d) Write down Φ_{cs} from the paper, and <u>explain</u> its physical significance.

e) Show that if there is only a simple interface (no layering, which means $M = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$) that T and R reduce to what we solve for in class. Show it works for <u>both</u> TE and TM waves.

3) Using the Mathematica code, create a 1/4 wave stack consisting of a glass (n=1.5) cover and substrate, and SiN layers (n=1.8) inside. Make the 1/4 wave stack optimal for normal incidence and yellow light ($\lambda_0 = 580\,nm$). Remember $\lambda_{glass} = \frac{\lambda_0}{n}$, etc.

a) Calculate R, T for m = 2, 10, 100, where m is the number of repeats of SiN / Glass.

b) Find m where R is first above 90%.

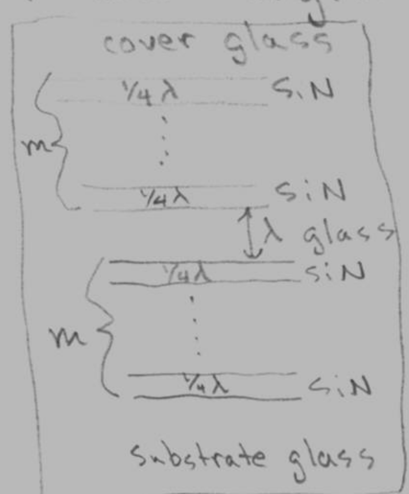
c) For the m in part (b), create a graph of R, T (on the same axis) as you vary the wavelength (without changing thicknesses) from 300 nm to 800 nm (the whole visible spectrum). Is this a good reflector for the whole range (could it be a mirror?)?

d) For the m in part (b), create a graph of R, T varying the incident angle ($\lambda_0 = 580\,nm$ again) from 0°-90°. Does this mirror work well off-axis?

4) a) Using the 1/4 wave stack from (3), find m for 99% reflection.

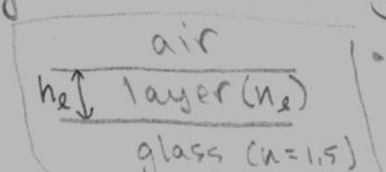
b) Now create two of those stacks on top of each other. Basically make m=2m. What is R now?

c) Do the same as in (b), but this time make a space between the first m stacks and the second m stacks that is not 1/4 wavelength, but 1 wavelength.



What is R and T now? What you have just made is a resonant cavity. We'll learn more about that later.

5) a) Say you have a glass substrate and you want to eliminate reflection at normal incidence with a single layer on the glass. So your structure looks like



. Using your code, solve for (λ_0=580nm)

M in terms of constants, n_e and h_e. Then set $r_{cs}=\emptyset$ to find at least one pair of n_e and h_e. Use normal incidence. This is an anti-reflection coating.

b) Make a graph for one case with set λ_0 varying θ_c (0→90°).

5) c) Using what you know of resonators, if the reflection coefficient for the interface between the air ($n = 1$) and n_e equals the reflection coefficient for the interface between n_e and the glass layer, you can make a resonant cavity out of the n_e layer. At resonance, $R_{tot} = 0$, which is what we want.

Solve analytically for n_e in terms of n_{glass} so that $R_{air,n_e} = R_{n_e,glass}$ (again at normal incidence).

d) For free space wavelength, λ_0, what thicknesses will yield resonance, and hence zero reflection? Solve in terms of λ_0 and n_{glass}. Now you have the answer analytically.

6) In (3) and (4) we talked about ¼ wave stack mirrors. Another way to make a mirror is with smooth metal. Consider the situation where you have a glass ($n=1.5$) substrate with a thin metal layer (set $n = -5i$). Just make the cover air.

a) How thick must the metal be for incident light at normal incidence and wavelength $\lambda_0 = 580\text{nm}$ for the reflectance to be 99%?

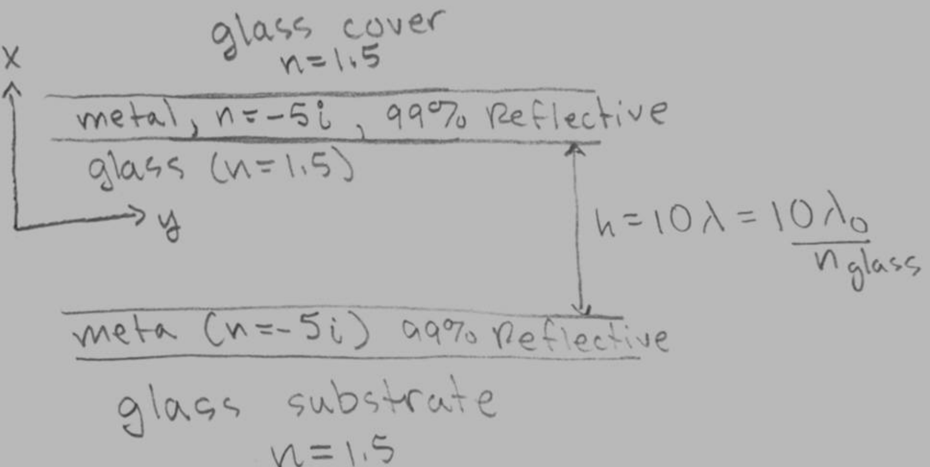
b) Plot R(thickness) from ∅ thickness to the thickness in part (a). {R = reflection coefficient}.

c) Plot R as you vary λ_0 from 300nm to 800nm still with normal incidence and using the thickness from part (a).

d) Plot R as you vary $\theta_{incident}$ from 0° to 90° at $\lambda_0 = 580\text{nm}$.

e) How does this mirror perform compared to the one in problem (3)?

7) Using the metal from problem (6), and the thickness from (6)(a), make the following structure



a) Plot R as you vary the incident wavelength λ_0 from $\lambda_0 = 580nm - 2\Delta\lambda$ to $\lambda_0 = 580nm + 2\Delta\lambda$ where $\Delta\lambda$ is the distance between successive peaks. (you should show 5 peaks with 2 right at each edge). What is $\Delta\lambda$? Check your result using the wikipedia article on Fabry-Perot resonators.

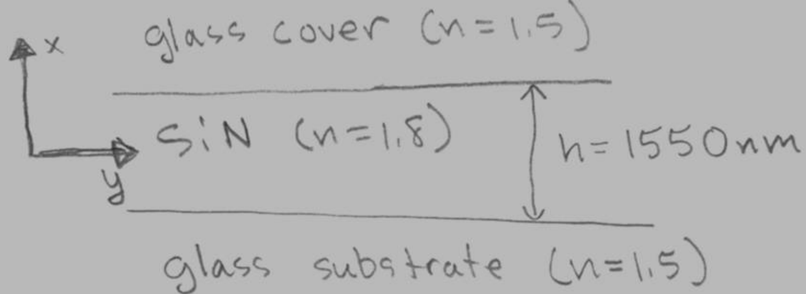
b) Increase h to 20λ, and redo your graph from part (a). Also refind $\Delta\lambda$.

c) Using the configuration from part (a) and $\lambda_0 = 580nm$, plot $R(\theta_{incident})$ vary $\theta_{incident}$ from $0° \to 90°$.

d) For the configuration from (a) at $\lambda_0 = 580nm$ and $\theta_{incident} = 0°$, plot uu^* in the resonator and a few wavelengths outside above and below (as a function of x). Set u(at substrate) = 1.

e) Change the thickness of the mirrors so their reflectance is now only 90%, and remake the plot from part (a). What changed?

8) Set up the situation below



$\lambda_0 = 1550\,\text{nm}$

a) Plot $\chi_m(\beta)$ for this structure. $\{1.5 \le \beta \le 1.8\}$

b) How many bound modes are there?

c) Find β for each of the modes from part (b).

d) On the same axis, plot $u(x)$ for each of the modes. Set u at one of the boundaries equal to 1.

e) What is the cut-off h for the second order mode? I.e. as you decrease h, at what point is there only one bound mode? ($\lambda_0 = 1550\,\text{nm}$).

f) Where $h = 1550\,\text{nm}$, what is the cut-off λ_0 for the second order mode? Note: usually they talk about cut-off frequency (given by $f = c/\lambda_0$).