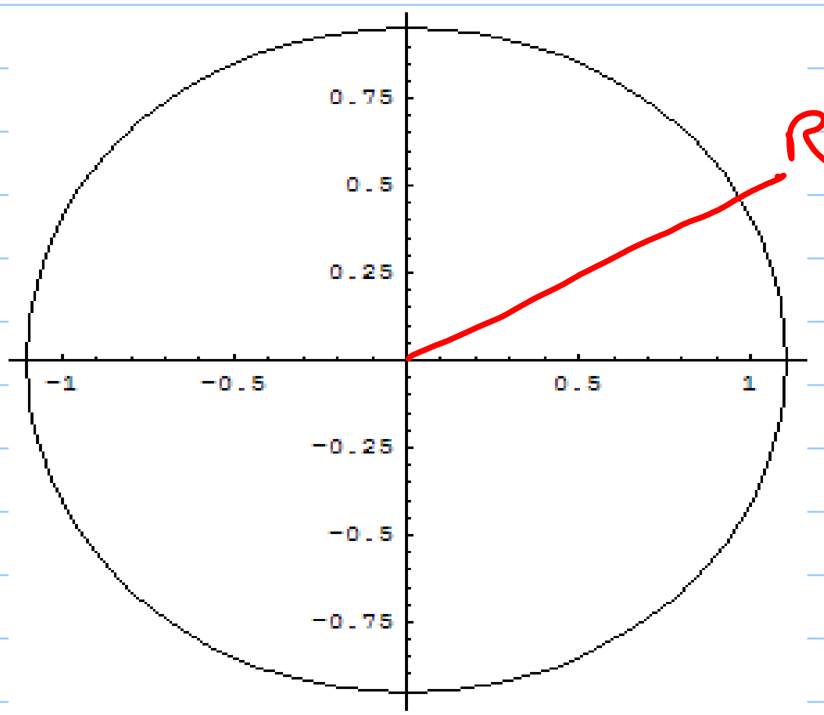
12/6/06

Note Title 12/6/2006

Due to rotation the Earth is flattened at the poles.



one way to approximate this is to model the grav. potential as

$$\psi(r=R,\theta,\varphi) = V_0(1 - J_2 P_2(\cos\theta))$$

$$J_2 \ll 1$$

Axially symmetric, so

$$\psi(r,\theta) = \sum_{l=0}^{\infty} A_l r^l + B_l r^{-(l+1)}) P_l(\cos\theta)$$

B.C.

$$\psi(r=R,\theta) = V_0\left(1 - J_2 P_2 \cos(\theta)\right)$$

$$= \sum_{l=0}^{\infty} \left(A_l R^l + B_l R^{-(l+1)}\right) P_l(\cos\theta)$$

NB $1 = P_0$

So

1) multiply both sides by P_0 and integrate

$$\int_{-1}^{1} \underbrace{V_0 P_0}_{V_0} - \underbrace{\bar{J}_2 P_0 P_2}_{0} \, d(\cos\theta) =$$

$$\sum_{l=0}^{\infty} \left(A_l R^l + B_l R^{-(l+1)} \right) \underbrace{P_0 P_l}_{\delta_{l0}}$$

$$\Rightarrow \boxed{V_0 = A_0 + B_0 R^{-1}}$$

$$V_0\left(1 - J_2 P_2 \cos(\theta)\right)$$

$$= \sum_{\ell=0}^{\infty} \left(A_\ell R^\ell + B_\ell R^{-(\ell+1)}\right) P_\ell(\cos\theta)$$

2) Do the same for P_2

$$-V_0 J_2 \underbrace{\int_{-1}^{1} P_2 P_2 \, d(\cos\theta)}_{\frac{2}{2\ell+1} = \frac{2}{5}}$$

$$= \sum_{\ell=0}^{\infty} \left(A_\ell R^\ell + B_\ell R^{-(\ell+1)}\right) \underbrace{P_2 P_\ell}_{\frac{2}{5}\delta_{2\ell}}$$

$$-V_0\, \mathcal{J}_2 \left(\cancel{\tfrac{2}{5}}\right) = A_2 R^2 + B_2 R^{-3} \left(\cancel{\tfrac{2}{5}}\right)$$

$A_2 \Rightarrow 0$ why?

$$-V_0\, \mathcal{J}_2 = B_2 R^{-3}$$

$$\Rightarrow \boxed{B_2 = -V_0\, \mathcal{J}_2 R^3}$$

Recall

$$\boxed{V_0 = A_0 + B_0 R^{-1}}$$

we ignore const poten.

$$\boxed{B_0 = V_0 R}$$

$$\Psi(r,\theta) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + B_\ell r^{-(\ell+1)}\right)P_\ell(\cos\theta)$$

$$= V_0\frac{R}{r} - V_0 J_2\left(\frac{R}{r}\right)^3 P_2$$

$$\boxed{= V_0\frac{R}{r}\left[1 - J_2\left(\frac{R}{r}\right)^2 P_2(\cos\theta)\right)}$$

$\lim_{J_2\to 0}$ we recover the standard result for a spherical earth.