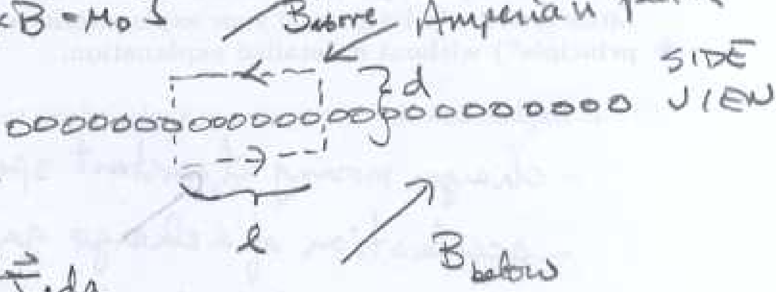
4. Derive the boundary condition on parallel component of the magnetic field across a sheet of current. Start with one of Maxwell's equations. $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

Stokes theorem



$$\int \vec{\nabla}\times\vec{B}\cdot d\vec{a} = \mu_0 \int \vec{J}\cdot d\vec{a}$$

$$\oint \vec{B}_{\parallel}\cdot d\vec{l} = \mu_0 I_{enclosed}$$

turns/meter

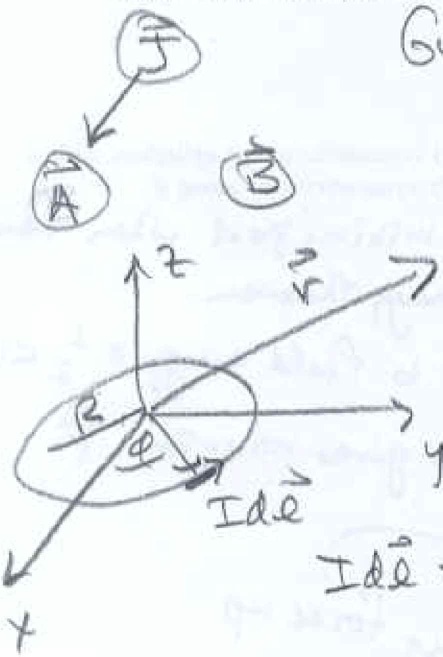
$\vec{B}_{below}\cdot\vec{d} = 0$ since $d \to 0$

$$B^{\parallel}_{below}\, l - B^{\parallel}_{above}\, l = \mu_0 \, n I_0 \, l$$

Amps/turn

$$\boxed{B^{\parallel}_{below} - B^{\parallel}_{above} = \mu_0 n I_0}$$

5. A wire loop carrying current I_0 of radius R is centered in the x-y plane. Derive an integral expression for the vector potential at an arbitrary point that I can directly put into Mathematica to evaluate.



Given $\vec{J}$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\, d\tau'}{|\vec{\mathscr{r}}|} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l}}{|\vec{\mathscr{r}}|}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r}\,' = R\cos\varphi'\hat{x} + R\sin\varphi'\hat{y} + 0\hat{z}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$$

$$|\mathscr{r}| = \sqrt{(x - R\cos\varphi')^2 + (y - R\sin\varphi')^2 + z^2}$$

$$I d\vec{l} = I_0 R\, d\varphi'\, \hat{\varphi} = I_0 R\, d\varphi' \left(-\sin\varphi'\hat{x} + \cos\varphi'\hat{y}\right)$$