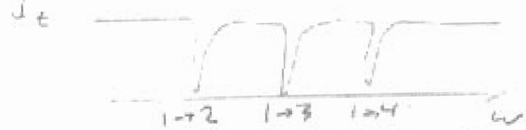
How do we get gain for photons?

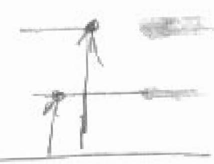
atom energy levels

observe: pass white light through medium, initially in ground state
record transmitted spectrum:



assuming all are allowed.

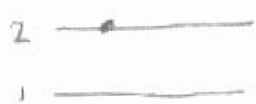
→ absorption



e^- → excited state

note - each line has a finite width

Now start w/ e^- in excited state:



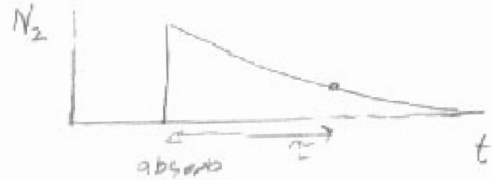
N_2 = population of level 2

e^- can decay to ground state
- if radiative → spontaneous emission
- non-radiative → generally heats medium

rate eqn: $\left(\frac{dN_2}{dt}\right)_{sp} = -AN_2$ spont. em. coeff = A

$1/A$ = lifetime

$\left(\frac{dN_2}{dt}\right)_{nr} = -\frac{N_2}{\tau_{nr}}$ τ_{nr} = non-radiative lifetime



Rate eqn for ground state

$$\left(\frac{dN_1}{dt}\right) = +AN_2 + \frac{1}{\tau_{nr}} N_2$$

Absorption rate eqn

$$\left(\frac{dN_1}{dt}\right)_a = -W_{12} N_1$$

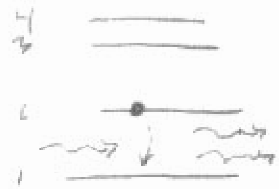
W_{12} transition rate $1 \to 2$

$= \sigma_{12} F$

σ_{12} absorption cross section

F photon flux

Now shine light on medium when it is in its excited state:



1) can now absorb from $2 \to 3$, $2 \to 4$

2) if $E_{phot} = \hbar\omega_{21}$ → stimulated emission

outgoing photon shares phase and direction of input photon
→ gain, amplification

$$\left(\frac{dN_2}{dt}\right)_{se} = -W_{21} N_2$$

Is $W_{12} = W_{21}$? If there are degeneracies, higher rate if more states are available.

$$g_1 W_{12} = g_2 W_{21} \quad \text{or} \quad g_1 \sigma_{12} = g_2 \sigma_{21}$$

Net effect on the beam:

F - photon flux number/sec/area

beam traverses material:

S - area $S\,dz$ = volume of slice

dz = element along z

each interaction will add or subtract a photon from beam

$$S\,dF = (W_{21}N_2 - W_{12}N_1)\,S\,dz \qquad N\text{'s} = \#/\text{volume}$$

or
$$\frac{dF}{dz} = W_{21}N_2 - W_{12}N_1 \qquad \begin{matrix} <0 \text{ absorb} \\ >0 \text{ amplify} \end{matrix}$$

require a population inversion for gain:

$$W_{21}N_2 - W_{12}N_1 = W_{21}\left(N_2 - \frac{g_2}{g_1}N_1\right) > 0$$

$F(z)$ inside medium:

$$F(z) = F_0\,e^{gz} \qquad \text{with } g = W_{21}\left(N_2 - \frac{g_2}{g_1}N_1\right)$$

for a fixed length: $z = L$

$$F_{out} = F_0\,e^{gL} = F_0 G_0$$

G_0 = small-signal single-pass gain

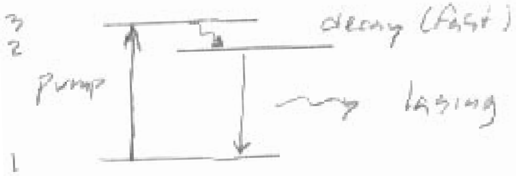
Amplifier:

$$F_{out} = G_0^n F_0 \qquad n = \#\text{ passes.}$$

with $G_0 = 10$, only 6 passes to go from 1 nJ to 1 mJ

cannot go indefinitely b/c of limited stored energy

Pumping: 3-level vs 4-level



3-level

pump hard: deplete level 1 to obtain inversion, avoid absorption 1→2

4-level

if level 2 is not populated, any population in 3 → inversion.

∴ higher gain

pump bands: often many potential pump bands:

→ broadband absorption.

eg. Nd: YAG many lines, allows pumping by flashlamps

$\hbar\omega_{pump} - \hbar\omega_{lase}$ = energy left in medium.

Ti:sapphire: $\frac{532\text{ nm}}{800\text{ nm}}$ = max fraction of pump power that can be extracted

Yb:KGW $\frac{980\text{ nm}}{1030\text{ nm}}$ better (quasi 3-level)

Can see that lamp pumping is less efficient, more heat

laser pumping is best.

→ diode pumped lasers