MATH-300 - Foundations of Adv. Mathematics **Field 2009**
Final Exam **Due: Thursday June 18, 5:00pm**

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1. Using induction, prove that for each even natural number n,

$$\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\cdots\left(1-\frac{(-1)^n}{n}\right)=\frac{1}{2}$$

2. Let $f : X \to Y$, and let $\hat{F} : \mathcal{P}(Y) \to \mathcal{P}(X)$ be defined by

$$\hat{F}(B) = \{x \in X \mid f(x) \in B\}.$$

Prove that $\hat{F}$ is one to one if and only if f maps onto Y.
(Note: $\mathcal{P}(X)$ is the power set of X)

3. For each of the following diophantine equations, either show that no solutions exist or find a solution.

   (a) $17x + 13y = 100$

   (b) $21x + 14y = 147$

4. Consider the identity: For $0 \leq m \leq k \leq n$,

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$$

Provide 2 proofs of this identity:

   (a) Give a direct proof using the definition of combinations.

   (b) Give a combinatorial proof

5. Provide a LaTeXversion of your resume (or curriculum vitae). It should include contact information, employment history and educational history (at a minimum). Please keep the resume to a single page.