Lecture 37

Homework due Monday April 21
Exam Wednesday April 23

Shadowitz chapter 7

Learning objectives for exam 4 are that you will be able to:

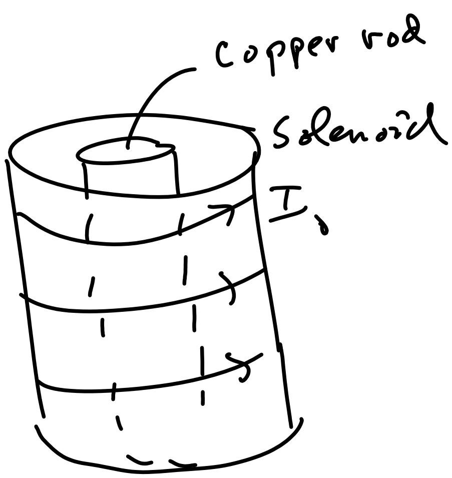
-be able to calculate the magnetic field due to a time changing electric field.
-be able to apply conservation of energy in integral form (starting from the work-energy theorem going to Poynting's theorem) to a simple system.
-be able to calculate the bound charge given the electric dipole moment per volume.
-be able to calculate the bound current given the magnetic dipole moment per volume.
-be able to apply perturbative methods to determine the effect of feedback on a system when it is described using a simple block diagram.
-be able to apply Gauss's and Ampere's laws (expressed with D and H) in a simple system to find E and B.

You will be given the triangle diagrams, Poyntings theorem, and the defns related to electric fields in matter.

The problem we face is finding M (just like finding P for electric fields)

Assume linear magnetic material

You might expect $M = \chi_m B$



$$B_{Solenoid} = \mu_0 n I_0 = \mu_0 K_0$$

$$\vec{B}_{copper} = \mu_0 \vec{K}_b = \mu_0 \vec{M} \times \hat{n}$$

what about the sign of the bound current?

$$B_{tot} = B_{Solenoid} + B_{copper}$$

Maxwell's eqn $\quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} = \mu_0 \left(\vec{J}_f + \vec{J}_{bound} \right)$

$$\vec{J}_{bound} = \vec{\nabla} \times \vec{M}$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \vec{\nabla} \times \vec{M} = \vec{J}_f$$

$$\vec{\nabla} \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M} \right)}_{\vec{H}} = \vec{J}_f$$

Amperes's law in $\vec{H}$

$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

Linear material $\quad \vec{M} = \chi_m \vec{H}$ $\quad$ positive for paramagnetic and negative for diamagnetic stuff

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{\vec{B}}{\mu_0} - \chi_m \vec{H}$$

$$\underbrace{\mu_0 (1 + \chi_m)}_{\mu} \vec{H} = \vec{B}$$

μ $\quad$ permeability

Method to find B using Ampere's law, written in terms of H, in a symmetrical current distribution in a linear material:

(1) apply Ampere's law using a path in the region where you want to know B.

(2) solve for H then get B by $\vec{H} = \mu \vec{B}$

(3) solve for M using H and get the bound currents from

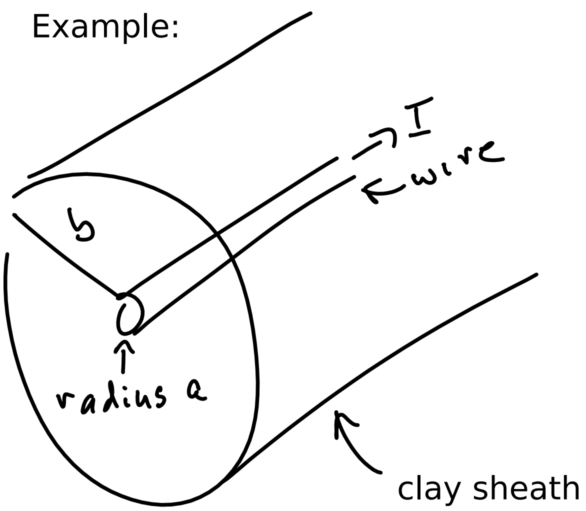
$$\vec{K}_b = \vec{M} \times \hat{n}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

informational: Why introduce H?

H is assocated with free currents which we can control with our power supplies. We cannot change the bound currents except by applying a field and having the material respond.

Example:



Find B and bound currents in the clay.

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

congruous: How do I calculate H? What do we need to know to apply Ampers's law?

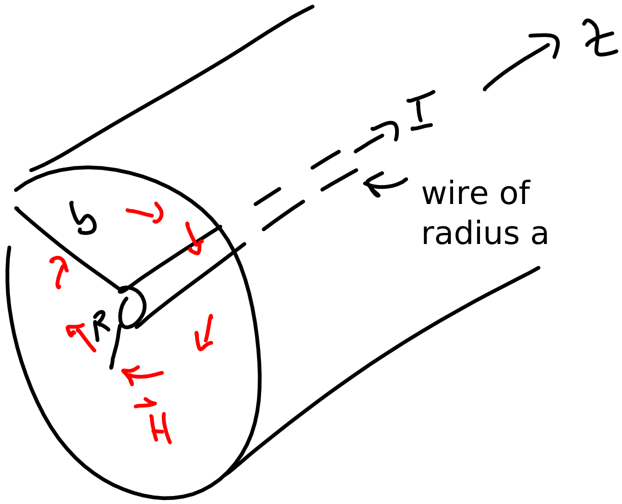
$$\vec{\nabla} \times \vec{H} = \vec{J}_f$$

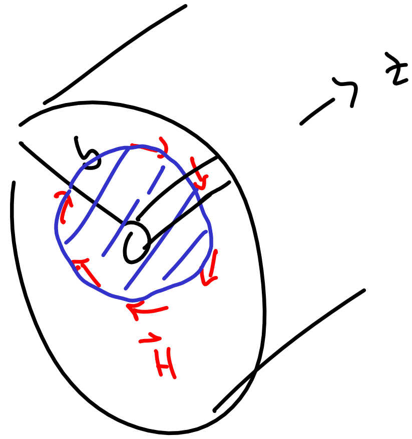
$$\downarrow$$

$$\int \vec{\nabla} \times \vec{H} \cdot d\vec{a} = \int \vec{J}_f \cdot d\vec{a}$$

$$\downarrow$$

$$\oint \vec{H} \cdot d\vec{r} = I_f$$





$$\oint \vec{H}\cdot d\vec{r} = H\,2\pi R = \int \vec{J}\cdot d\vec{a}$$

$$\vec{H} = \frac{I}{2\pi R}\,\hat{\varphi}$$

$$\vec{B} = \mu \vec{H} = \frac{\mu_0 (1+\chi_m) I}{2\pi R}\,\hat{\varphi}$$

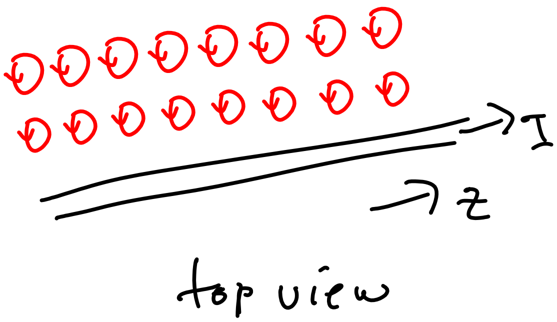
$$\vec{M} = \chi_m \vec{H} = \chi_m \frac{I}{2\pi R}\,\hat{\varphi} \qquad \text{let } R = r$$

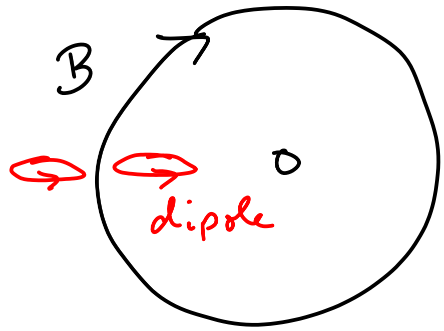
$$\vec{J}_b = \vec{\nabla}\times\vec{M} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\chi_m I}{2\pi r}\right)\hat{z} = 0$$

$$\vec{K}_b = \vec{M}\times\hat{n} = \begin{cases} \dfrac{\chi_m I}{2\pi a}\,\hat{z} & \text{at } r = a \\[2ex] -\dfrac{\chi_m I}{2\pi b}\,\hat{z} & \text{at } r = b \end{cases}$$

incongruous: It looks like there is a net bound current; the difference between the currents on the two surfaces. This must be wrong because the bound current is made of little closed loops. How can you have a current left over?

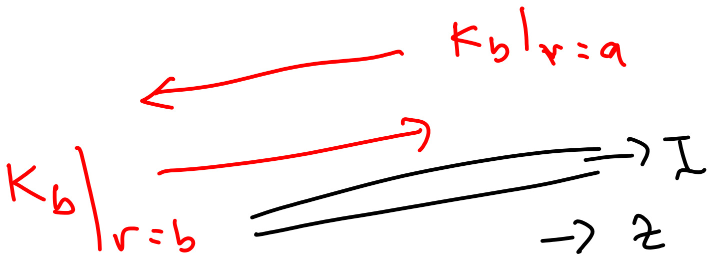
The current on the inner surface per length L of the rod is

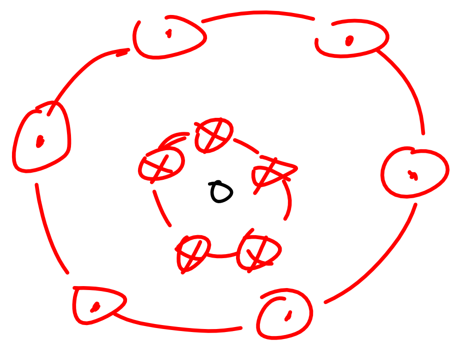




End view

interior bound currents cancel leaving only two surface currents.





End view

Top view

$$I\big|_{r=a} = K_b\big|_{r=a} 2\pi a = \frac{\chi_m I}{2\pi a}\hat{z}\, 2\pi a = \chi_m I\,\hat{z}$$

$$I\big|_{r=} = K_b\big|_{r=} 2\pi b = -\frac{\chi_m I}{2\pi b}\hat{z}\, 2\pi b = -\chi_m I\,\hat{z}$$

Homework problem 5.) A rod of copper (radius b) is placed in a solenoid n turns per length and current I0. What is B inside the copper? What is M and what are the bound currents?

Homework problem 6.) A thermonuclear weapon was detonated underground 30 years ago. The radioactive lifetime of the remants is thousands of years so the heat energy generated can be accurately modeled as being steady. It can be shown that the PDE for the steady state temperature distribution in the Earth is

$$k\nabla^2 T = g$$

where k is the thermal conductivity and g is the heat source energy per volume.

This is Poisson's equation. We can model the surface of the Earth above the detonation as not allowing any thermal energy to flow out of it.

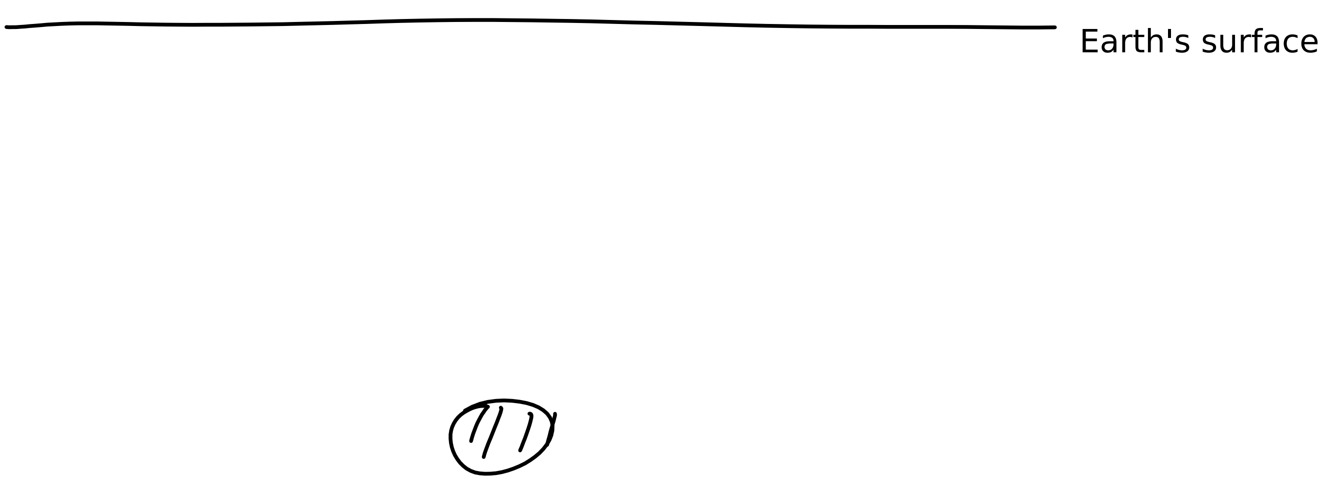
(a) what parameters in electrostatics are the analogs of T and g?

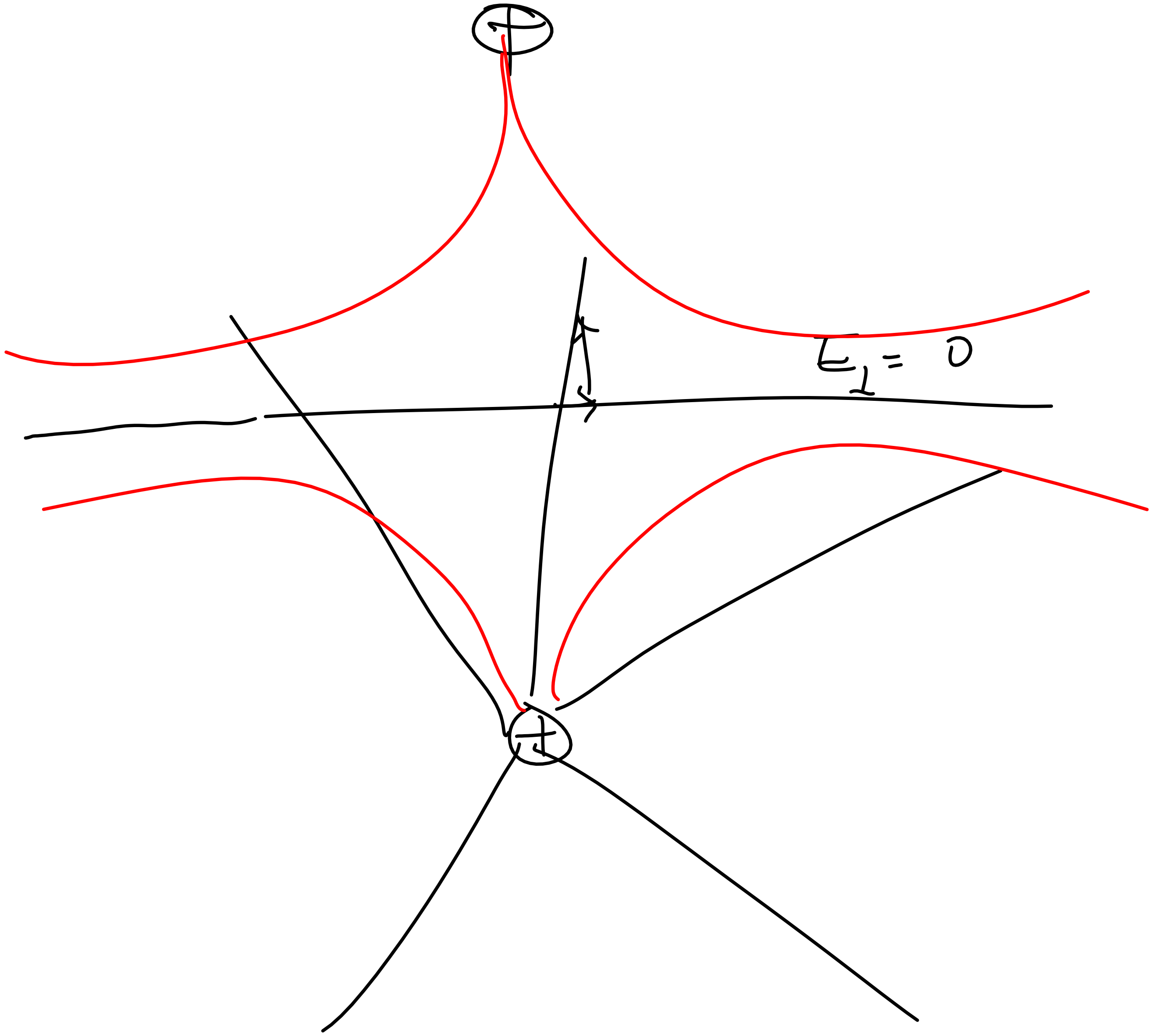
(b) the flow of heat energy per time per area h is

$$\vec{h} = -k\vec{\nabla} T$$

What is the analog of h in electrostatics?

(c) sketch the electrostatic analog (E lines and V contours) to this problem which satisfies the boundary conditions. Sketch also the thermal solution with lines of equal temperature and vector field lines of h.




$L_1 = 0$