10-8-07 Review

Note Title 10/8/2007

Main topics

Geometric Series
finite + infinite

Power series
McClaurin + Taylor

Euler's formula
$\sin(iy)$, $\cos(iy)$

Powers + roots of complex #

$x+iy \rightarrow Re^{i\varphi}$

$Re^{i\varphi} \rightarrow x+iy$

Solve $Ax=y$ by Gaussian elimination

Be able to tell when
1) no solution exists
2) a unique solution ex
3) infinitely many

Null space of a matrix
Linear in dependence

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$ no solution

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$ mathematica says $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ but Det $= 0$
null space ?

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$ $\begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$ unique since Det $\neq 0$

null space satisfies

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x = -2y$$

$\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ spans null space

So any solution can be written

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

eigenvalues
eigenvectors

Rotations
Reflections

Know what it means for a matrix to be

1) symmetric
2) orthogonal
3) Hermitian
4) unitary

understand that not all invertible matrices are diagonalizable

Diagonalization is a coordinate transformation

if $C^{-1}MC = D$ then

D represents the linear operator in the new coordinates

Linear operator

M

$$\begin{bmatrix} m_{11} & m_{12} & & \\ & & \ddots & \\ & & & m_{NN} \end{bmatrix}$$

coordinate system 1

$$\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix}$$

coordinate system 2

$$C^{-1} M C = D$$