MATH348-Advanced Engineering Mathematics		**Short Homework Six**
	FOURIER SERIES	
Text: 11.1-11.3	Lecture Notes: 9	Lecture Slides: N/A

Quote of Short Homework Six	
Darth Vader: Don't be too proud of this technological terror you've constructed.	
	Star Wars : Episode IV (1977)

1. Goals

The goal of this assignment is to review the formula of Fourier series. After this assignment the student should:

- Understand how the Fourier coefficients are associated to the Fourier series.

2. Objectives

To achieve the previous goals the student will meet the following objectives:

- Read formula 6 on page 480, theorem 11.1.1 on page 482, formula 5 and 6 on page 487, theorem 11.3.1 on page 491 and formula 7 on page 497.
- Compute Fourier series based on given integrals.

3. Problems

Given the following integrals,

$$\int_{-\pi}^{\pi} f(x)dx = 0,\ \int_{-\pi}^{\pi} f(x)\cos(nx)dx = \frac{1}{n}\left[\sin(nx)|_0^\pi - \sin(nx)|_{-\pi}^0\right],\ \int_{-\pi}^{\pi} f(x)\sin(nx)dx = \frac{1}{n}\left[\cos(nx)|_{-\pi}^0 - \cos(nx)|_0^\pi\right],$$

$$\int_{-2}^{2} g(x)dx = 2,\ \int_{-2}^{2} g(x)\cos\left(\frac{n\pi}{2}x\right)dx = \frac{2x}{n\pi}\sin\left(\frac{n\pi}{2}\right)\Big|_0^1 - \frac{4}{n^2\pi^2}\cos\left(\frac{n\pi}{2}x\right)\Big|_1^2,\ \int_{-2}^{2} g(x)\sin\left(\frac{n\pi}{2}x\right)dx = 0,$$

$$\int_0^{2\pi} h(x)dx = \frac{8\pi^3}{3},\ \int_0^{2\pi} h(x)\cos(nx)dx = \left[\frac{x^2\sin(nx)}{n} + \frac{2x\cos(nx)}{n^2} - \frac{2\sin(nx)}{n^3}\right]_0^{2\pi},$$

$$\int_0^{2\pi} h(x)\sin(nx)dx = \left[\frac{-x^2\cos(nx)}{n} + \frac{2x\sin(nx)}{n^2} + \frac{2\cos(nx)}{n^3}\right]_0^{2\pi}$$

$$\int_{-\pi}^{\pi} m(x)dx = 2\pi,\ \int_{-\pi}^{\pi} m(x)e^{-inx}dx = -\frac{e^{-inx}}{in}\Big|_{-\pi}^{\pi}$$

3.1. **Fourier series: 2π-periodic.** Using the integral relations above find the Fourier series representation of $f(x)$. Simplify as much as possible.

3.2. **Fourier series: $2L$-periodic.** Using the integral relations above find the Fourier series representation of $g(x)$. Simplify as much as possible.

3.3. **Fourier series: Non-Standard Domain.** The Euler relations can be shown to hold on any 2π-domain. Consequently, the formula also hold for any $2L$-domain. Thus, if you are given a function on the domain (a, b) where $a < b$ and you are told that this function repeats itself every $b - a$ units then you can use the following formula,

$$a_0 = \frac{1}{2L}\int_a^b f(x)dx,\ a_n = \frac{1}{L}\int_a^b f(x)\cos\left(\frac{n\pi}{L}x\right)dx,\ b_n = \frac{1}{L}\int_a^b f(x)\sin\left(\frac{n\pi}{L}x\right)dx \tag{1}$$

where $2L = b - a$.[1] Using this idea find the Fourier series representation of $h(x)$. Simplify as much as possible.

3.4. **Fourier series: Complex.** Using the integral relations above find the complex Fourier series representation of $m(x)$. Simplify as much as possible.

[1] The take home message is this: If you take a reasonable (AKA square integrable) function and shove it into the Fourier coefficient formula and integrate over the principle period the Fourier series will have the affect of taking the graph and repeating it to the right and left of the principle period.