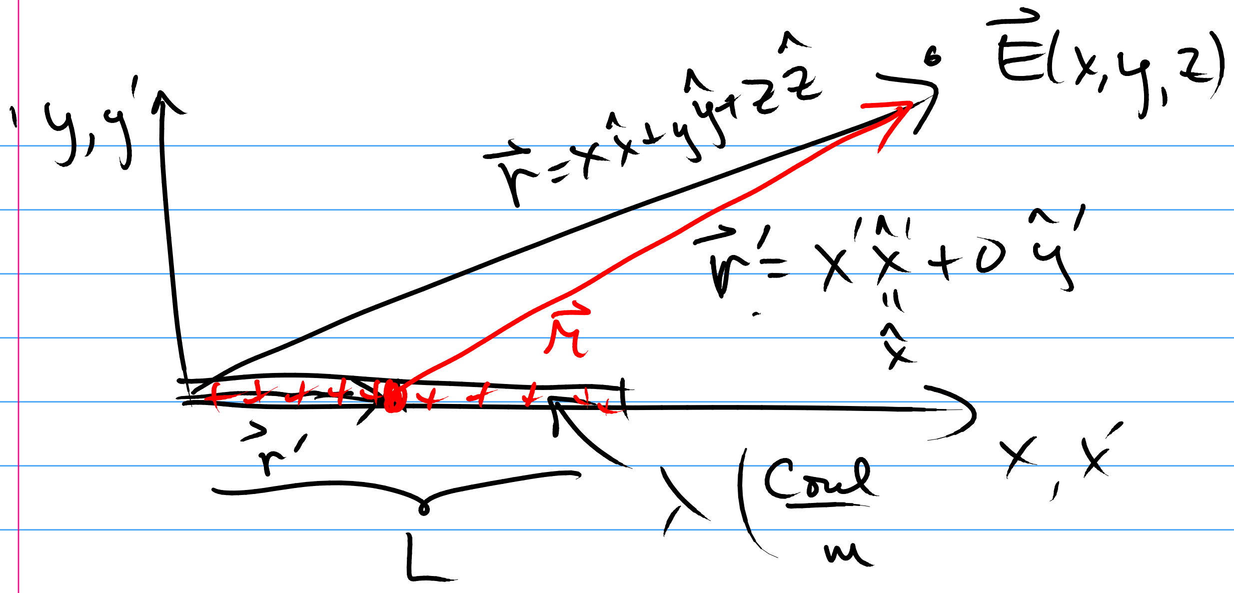


$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = (x - x')\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$|\vec{\mathscr{r}}| = \sqrt{(x-x')^2 + y^2 + z^2}$$

$$dq = \lambda\, dx'$$

$$\vec{E} = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dx'}{\left[(x-x')^2 + y^2 + z^2\right]^{3/2}} \left[(x-x')\hat{x} + y\,\hat{y} + z\,\hat{z}\right]$$

$$= \frac{\hat{x}}{4\pi\epsilon_0} \int_0^L \frac{\lambda\, dx'(x-x')}{\left[(x-x')^2 + y^2\right]^{3/2}} + \frac{\hat{y}}{4\pi\epsilon_0} \int_0^L \frac{\lambda\, y\, dx'}{[\quad]^{3/2}}$$

limiting case $x \gg x'$

$$E_x \propto \int_0^L \frac{\lambda\, dx'(x-x')}{\left[(x-x')^2+y^2\right]^{3/2}} \approx \int_0^L \frac{\lambda\, dx'\, x\left(1-\frac{x'}{x}\right)}{\left[x^2\left(1-\frac{x'}{x}\right)^2+y^2\right]^{3/2}}$$

$\frac{x'}{x} \to 0$

$$E_x = \int \frac{\lambda\, dx'\, x}{\left[x^2+y^2\right]^{3/2}}$$

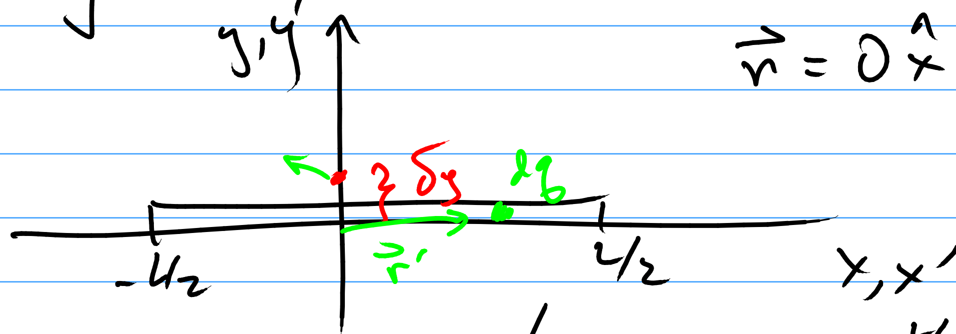
$$E_y = \int_0^L \frac{\lambda y\, dx'}{[\;]^{3/2}} \longrightarrow \int_0^L \frac{\lambda y\, dx'}{\left[x^2+y^2\right]^{3/2}}$$

$$\vec{E} = \frac{\hat{x}}{4\pi\epsilon_0}\int \frac{\lambda\, dx'\, x}{r^3} + \frac{\hat{y}}{4\pi\epsilon_0}\int \frac{\lambda\, dx'\, y}{r^3}$$

$$= \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda(x\hat{x}+y\hat{y})\,dx'}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{\int \lambda\, dx'}{r^2}\hat{r}$$

" Q

limiting case close to wire:



$$\vec{r} = 0\hat{x} + \delta y\,\hat{y}$$

$$\vec{E} = \cancel{\frac{\hat{x}}{4\pi\epsilon_0}\int_{-L/2}^{L/2}\frac{\lambda dx'(-x')}{\left[x'^2+\delta_y^2\right]^{3/2}}} + \frac{\hat{y}}{4\pi\epsilon_0}\int_{-L/2}^{L/2}\frac{\lambda\,\delta_y\,dx'}{\left[x'^2+\delta_y^2\right]^{3/2}}$$

symmetry

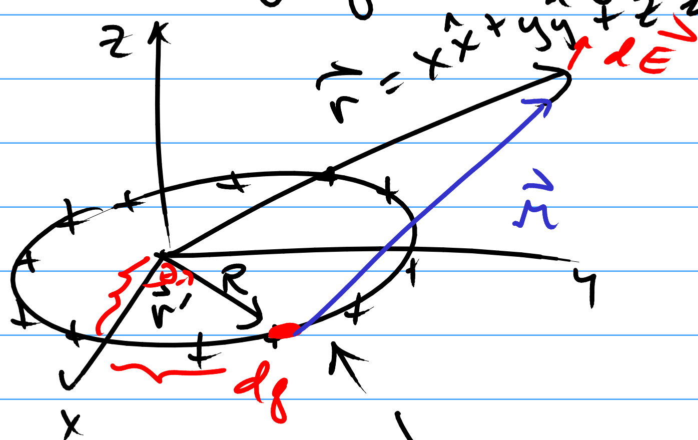
$$\frac{1}{4\pi\epsilon_0}\left[\frac{-4\lambda\delta_y}{L^2} + \frac{2\lambda}{\delta_y}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{1}{L}\left[-4\lambda\cancelto{0}{\left(\frac{\delta_y}{L}\right)} + 2\lambda\frac{1}{\delta_y/L}\right]$$

gets large

limit $\frac{\delta_y}{L} \to 0 \Rightarrow \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{\delta_y}$

Diff config of charge

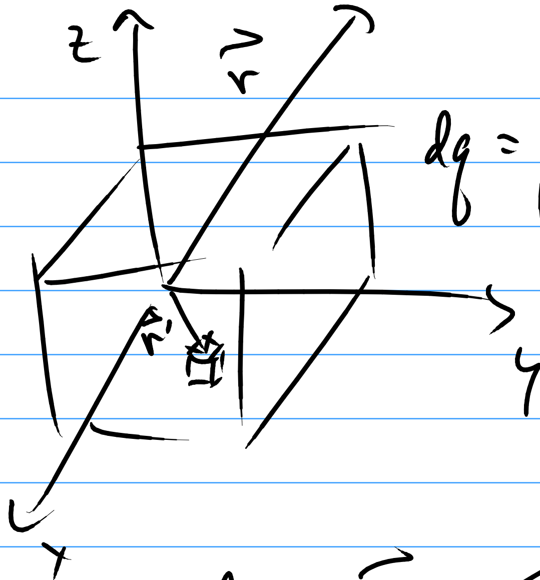


$$dq = \lambda R\,d\theta'$$

$$\int_0^{2\pi} R\,d\theta = 2\pi R$$

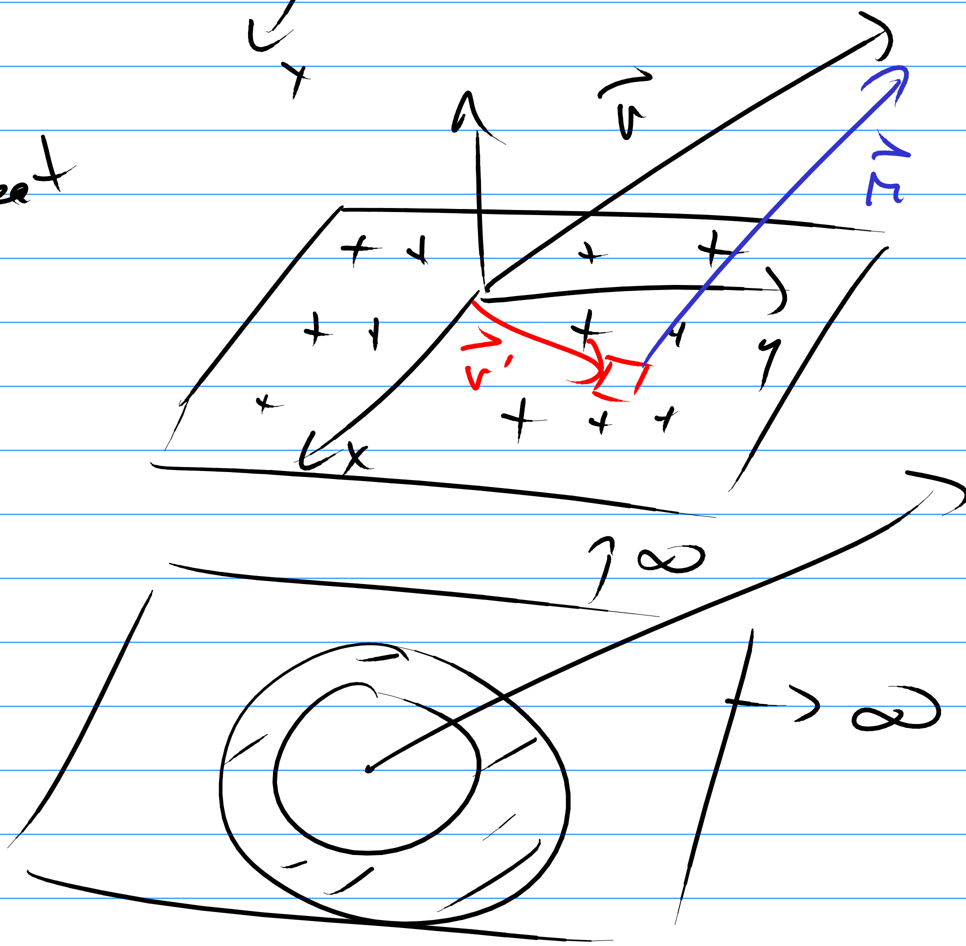
ring

Cube

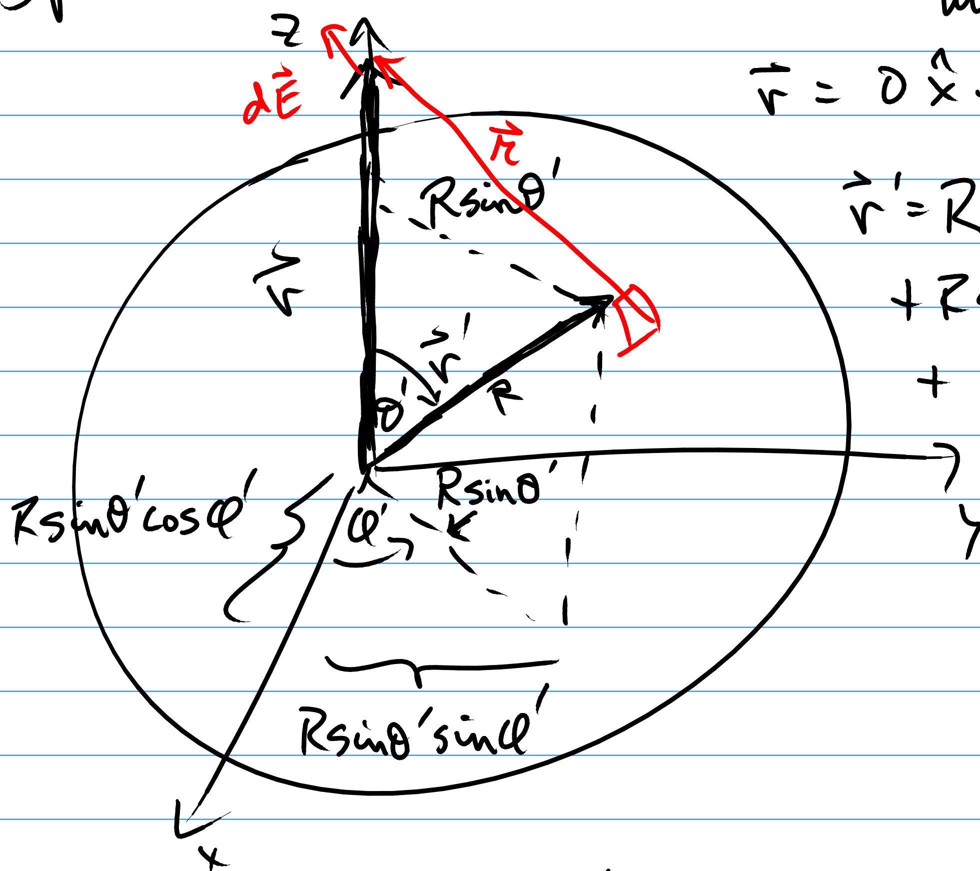


$dq = \rho \, dx' dy' dz'$

Sheet



Spherical shell $\quad \sigma$ const $\frac{C}{m^2}$



$$\vec{r} = 0\hat{x} + 0\hat{y} + z\hat{z}$$

$$\vec{r}' = R\sin\theta'\cos\varphi'\hat{x} + R\sin\theta'\sin\varphi'\hat{y} + R\cos\theta'\hat{z}$$

$$\vec{\eta} = \vec{r} - \vec{r}' = -R\sin\theta'\cos\varphi'\hat{x} + -R\sin\theta'\sin\varphi'\hat{y} + (z - R\cos\theta')\hat{z}$$

$$|\vec{\eta}| = \sqrt{\underbrace{R^2\sin^2\theta'\cos^2\varphi' + R^2\sin^2\theta'\sin^2\varphi'}_{R^2\sin^2\theta'(\cos^2\varphi' + \sin^2\varphi')} + (z - R\cos\theta')^2}$$

$$R^2\sin^2\theta'\underbrace{(\cos^2\varphi' + \sin^2\varphi')}_{1} + z^2 - 2zR\cos\theta' + R^2\cos^2\theta'$$

$$|\vec{\eta}| = \sqrt{R^2 + z^2 - 2zR\cos\theta'}$$ Law of Cosines