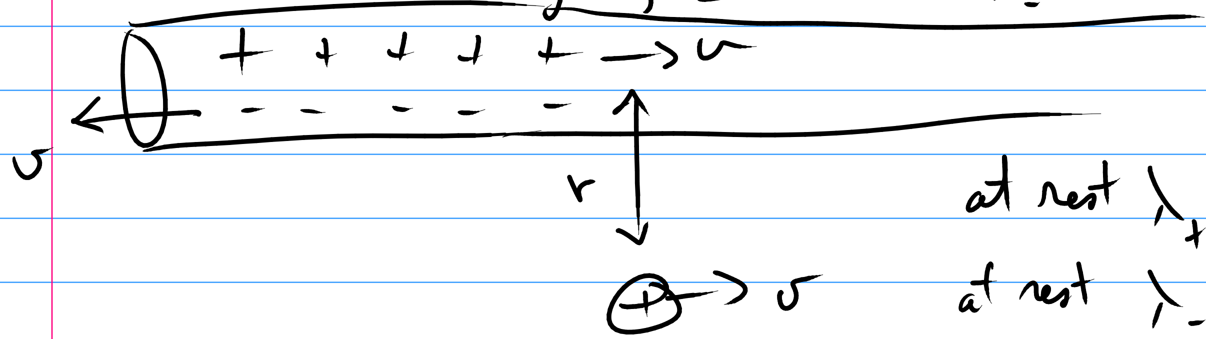
LAB FRAME

wire carrying $I = \lambda_+ v + \lambda_-(-v)$



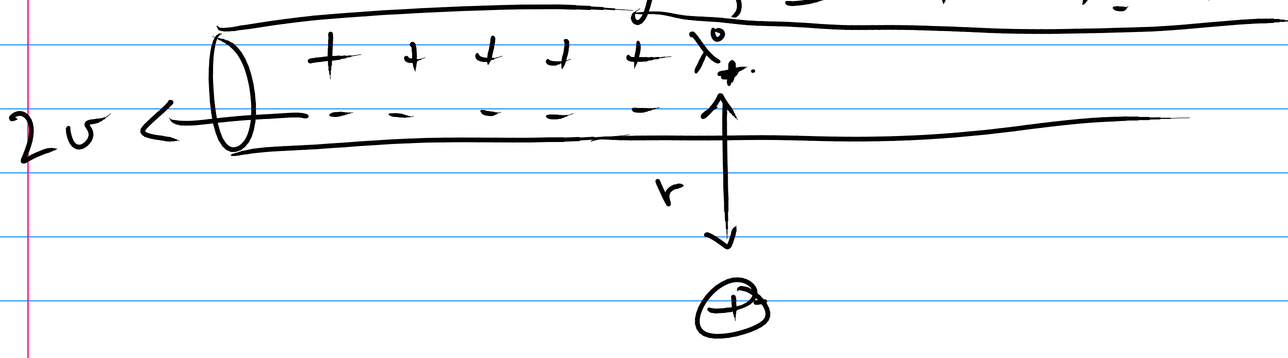
at rest $\lambda_+ = \lambda^0$.

at rest $\lambda_- = \lambda^0$

$$F = qvB = q v \frac{\mu_0 I}{2\pi r} \propto q v \frac{\mu_0 \lambda^0 v}{r} + \text{higher order terms in } v/c$$

CHARGE FRAME

wire carrying $I = \lambda_+(0) - \lambda_-(-2v)$



$$F = qE = q \frac{\lambda^0}{2\pi \epsilon_0 r} - q \frac{\lambda^0}{\sqrt{1-\left(\frac{2v}{c}\right)^2}} \frac{1}{2\pi \epsilon_0 r}$$

$$\frac{1}{\sqrt{1-\left(\frac{2v}{c}\right)^2}} \simeq 1 + \frac{4}{2}\frac{v^2}{c^2} + \cdots$$

$$F = \frac{q\lambda^{o}}{2\pi\epsilon_0 r}\left[1 - 1 - \frac{4}{2}\frac{v^2}{c^2} + \cdots\right] \propto \frac{\lambda^{o} q}{\epsilon_0 r}\frac{v^2}{c^2}$$

but $c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ or $\epsilon_0 = \frac{1}{\mu_0 c^2}$ so

$$F_{\text{electric field}} \propto \lambda^{o}\frac{q\,\mu_0 \cancel{c^2}}{r\,\cancel{c^2}} v^2 = \lambda^{o} q \frac{\mu_0 v^2}{r}$$

$$F_{\text{magnetic field}} \propto q v \frac{\mu_0 \lambda^{o} v}{r}$$