constant; ρ_1 is a function of x',

$$\rho_1 = g(x'),$$

where $g(x')$ is an arbitrary function of x'. In the original variables,

$$\boxed{\rho_1 = g(x - ct).} \tag{67.2}$$

To again verify that this really is the solution, we substitute it back into the partial differential equation 67.1. Using the chain rule

$$\frac{\partial \rho_1}{\partial x} = \frac{dg}{d(x-ct)} \frac{\partial (x-ct)}{\partial x} = \frac{dg}{d(x-ct)}$$

and

$$\frac{\partial \rho_1}{\partial t} = \frac{dg}{d(x-ct)} \frac{\partial (x-ct)}{\partial t} = -c\frac{dg}{d(x-ct)}.$$

Thus it is verified that equation 67.1 is satisfied by equation 67.2. Even though equation 67.1 involves both partial derivatives with respect to x and t, it can be integrated (in a coordinate system moving with velocity c). The general solution to equation 67.1 contains an arbitrary function, just like the examples of Sec. 65. In Sec. 69 we will derive this result in an easier way; we will not find it necessary to use the change of variables formula for partial derivatives.

Can the arbitrary function be determined in order to solve the initial condition? The general solution is

$$\rho_1(x, t) = g(x - ct),$$

but initially $\rho_1(x, 0) = f(x)$. Thus $f(x) = g(x)$. Consequently, the solution of the partial differential equation satisfying the initial condition is

$$\boxed{\rho_1(x, t) = f(x - ct),} \tag{67.3}$$

or equivalently

$$\boxed{\rho(x, t) = \rho_0 + \epsilon f(x - ct).}$$

If we move with the velocity c, the density stays the same. The density is said to **propagate** as a **wave** (called a **density wave**) with **wave speed** c. Note that this velocity may be different from the velocity at which an individual car moves.

Along the curves $x - ct =$ constant, the density stays the same. These lines are called **characteristics*** of the partial differential equation,

*The essential property of characteristics is that along those curves the partial differential equation reduces to an ordinary differential equation. Curves along which ρ is constant are not always characteristics (see Sec. 83).

$$\boxed{\frac{\partial \rho_1}{\partial t} + c\frac{\partial \rho_1}{\partial x} = 0.}$$

In this case the characteristics are all straight lines with velocity c, $dx/dt = c$. Sketching various characteristics in a space-time diagram, yields Fig. 67-3. Along each characteristic, the density equals the value it has at $t = 0$. Note that ρ_1 stays constant along the characteristic, but $\partial\rho_1/\partial t$ and $\partial\rho_1/\partial x$ may

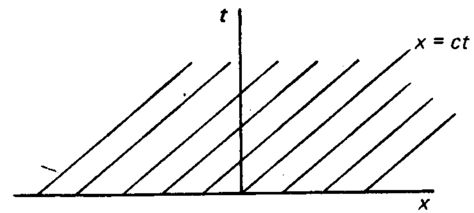

Figure 67-3 Characteristics of $\partial\rho_1/\partial t + c\,\partial\rho_1/\partial x = 0$.

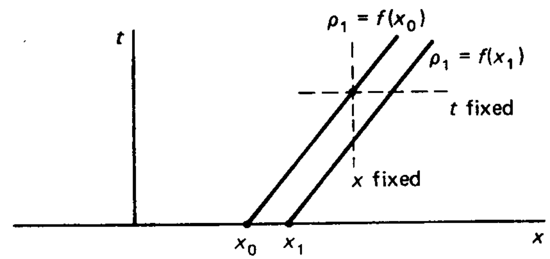

Figure 67-4 Density variations.

not be zero; see Fig. 67-4. As illustrated there $\partial\rho_1/\partial t$ may not equal zero since *keeping x fixed* ρ_1 may vary. Likewise $\partial\rho_1/\partial x$ is not necessarily zero since ρ_1 may change *keeping t fixed*. In Figs. 67-3 and 67-4 we have assumed $c > 0$. What is the sign of c? Recall

$$\boxed{c = \frac{dq}{d\rho}(\rho_0).} \tag{67.4}$$

The Fundamental Diagram of Road Traffic is of the form shown in Fig. 67-5. Thus the slope is positive for densities less than that corresponding to the capacity of the road and the slope is negative for densities greater than that corresponding to the road capacity. The sign of the slope is significant as we have indicated that small perturbations to a uniform density move at the