Linear Polarization Optics

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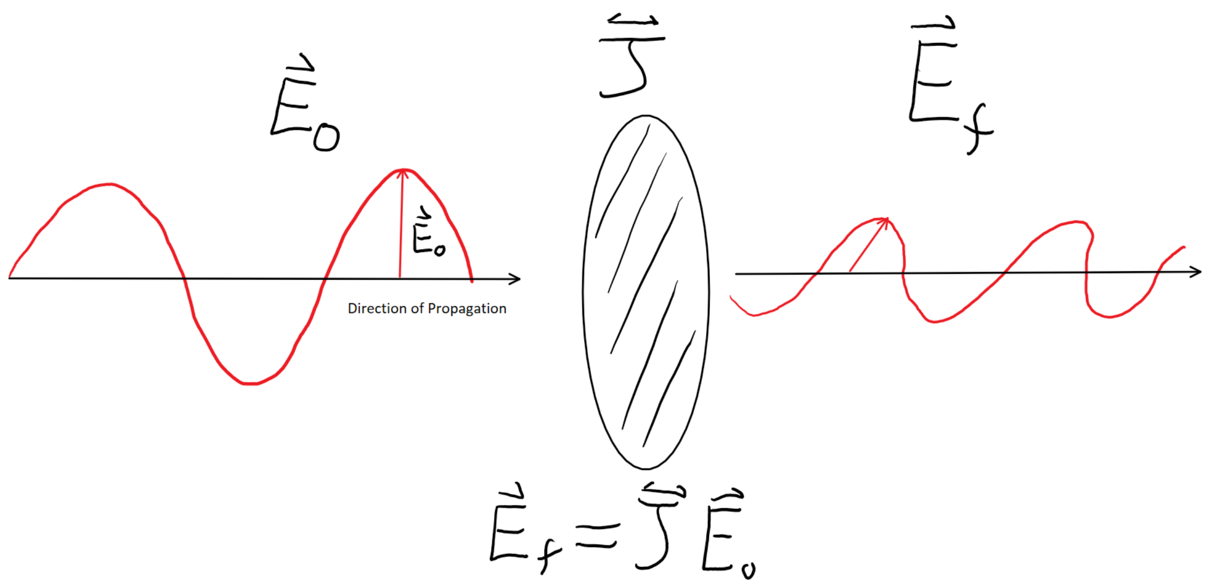
Polarization and electromagnetic (EM) waves in general can seem intimidating the first time you see them. Fortunately, there are notations and tricks that simplify the theoretical treatment a great deal. In the field of optics, Jones calculus is one method that concisely deals with polarization and the effect of some optical component on the polarization. While I don't expect you to know a great deal about polarization, or EM waves in general, using Jones calculus in the context of this experiment is as simple as matrix multiplication.

To start, we are assuming our electromagnetic field is in the form of plane waves,

$$\vec{E} = \overrightarrow{E_0}\cos(\omega t - kz)$$

where our wave is propagating in the z direction and $\overrightarrow{E_0}$ is the magnitude and polarization of our wave. $\overrightarrow{E_0}$ is the main concern of this lab and Jones calculus. Visually, we can draw the system as follows:



Since the EM wave is propagating along the z axis, $\overrightarrow{E_0}$ is on the x-y plane which we can represent by the Jones vector,

$$\overrightarrow{E_0} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

where $\sqrt{{E_x}^2 + {E_y}^2} = |\overrightarrow{E_0}|$. In other words we write $\overrightarrow{E_0}$ as a linear combination of plane waves pointing in the x and y directions.

To write the optical components, we need some operator (think matrix) that acts on the incident EM wave giving us the EM wave leaving the component.

$$\overrightarrow{E_f} = \overleftrightarrow{J}\,\overrightarrow{E_0}$$

Clearly $\overleftrightarrow{J}$ must be a 2x2 matrix since $\overrightarrow{E_f}$ and $\overrightarrow{E_0}$ are both 2x1 vectors. The form of the operator will depend on what direction of the linear polarizer's optical axis. For a linear polarizer with its axis at an angle θ with respect to the x axis the operator will look like,

$$\overleftrightarrow{J} = \begin{bmatrix} cos^2\theta & cos\theta sin\theta \\ cos\theta sin\theta & sin^2\theta \end{bmatrix}$$

For a detector that is not sensitive to polarization, this is the only matrix you need. For a detector that is polarization dependent, we can treat it as another operation. To simplify things, we setup our detector and emitter to be in the same direction, which we will call horizontally polarized. Then $\overrightarrow{E_0} = E_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and $\overleftrightarrow{J}_{detector} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. We can write the total expression for $\overrightarrow{E_f}$ as,

$$\overrightarrow{E_f} = \overleftrightarrow{J}\,\overrightarrow{E_0} = \overleftrightarrow{J}_{detector}\overleftrightarrow{J}_{polarizer}\overrightarrow{E_0} = E_0 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} cos^2\theta & cos\theta sin\theta \\ cos\theta sin\theta & sin^2\theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Thus our effective matrix for this system is $\overleftrightarrow{J}_{effective} = \begin{bmatrix} cos^2\theta & cos\theta sin\theta \\ 0 & 0 \end{bmatrix}$. From these matrices and the assumption that our emitter is linearly polarized, we expect the magnitude of our electric field at the detector to be $E_0 cos\theta$ for a detector with no polarization dependence and $E_0 cos^2\theta$ for a detector that is polarization dependent and aligned with the polarization of the emitter. Since $I = \frac{c\,n\,\varepsilon_0}{2}|E|^2$, we can see that the Jones vectors give the same result as Malus's Law: $I_f = I_0 cos^2\theta$.