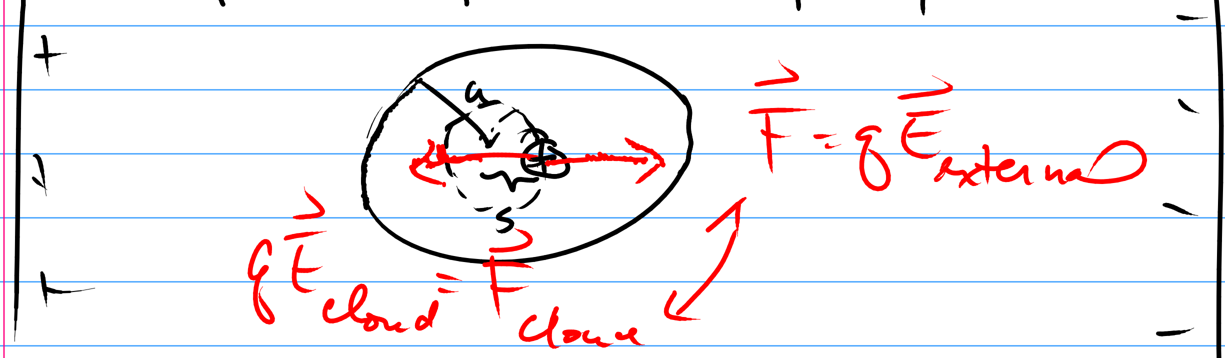
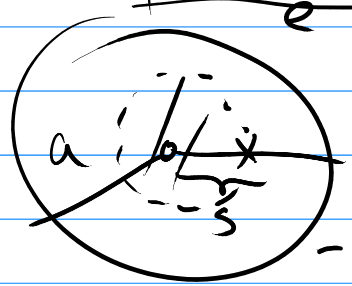
# INDUCED DIPOLE

Assume proton is a pt. particle




$$E_{cloud} = E_{external}$$



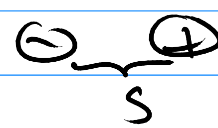

Gauss's law

$$E \, 4\pi s^2 = \frac{\rho \frac{4}{3}\pi s^3}{\epsilon_0}$$

$$E = \frac{\rho \, 4\pi s^3}{3 \, 4\pi s^2 \epsilon_0} = \frac{\rho s}{3\epsilon_0}$$

$$\rho = \frac{q}{\frac{4}{3}\pi a^3} \qquad E = \frac{q}{\frac{4}{3}\pi a^3} \frac{s}{3\epsilon_0}$$

$$E = \frac{q s}{4\pi a^3 \epsilon_0}$$

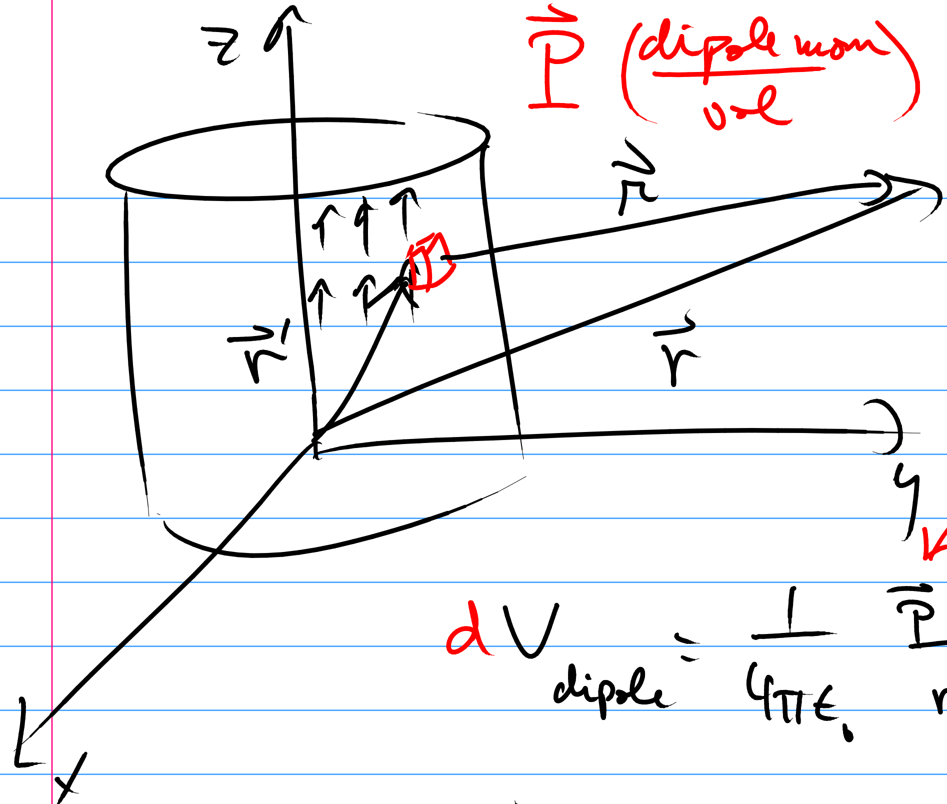



linear atom

$$p = qs = \alpha E_{applied} = \alpha \frac{qs}{4\pi a^3 \epsilon_0}$$

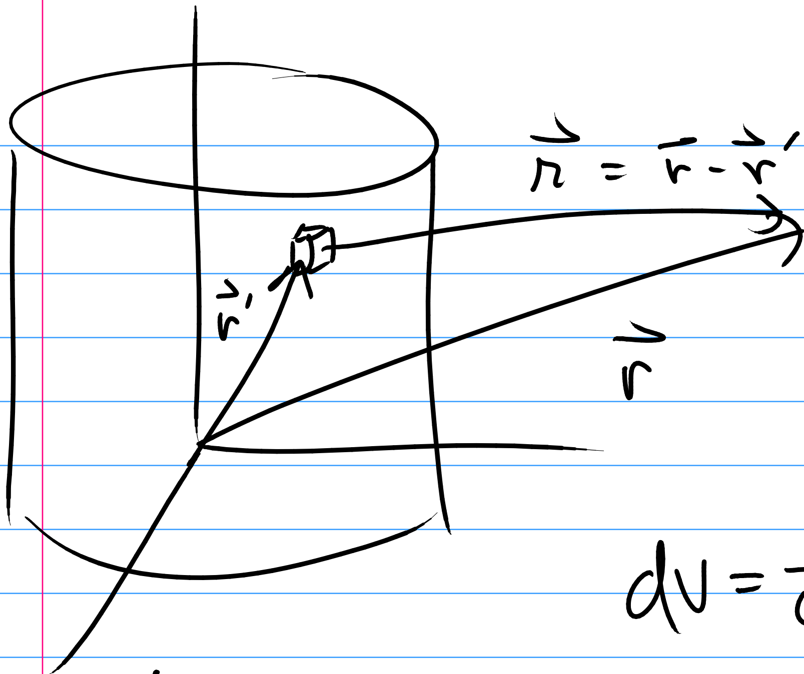
$$\alpha = 4\pi a^3 \epsilon_0$$

$\vec{P}\left(\frac{\text{dipole mom}}{\text{vol}}\right) \sim \rho\left(\frac{\text{kg}}{\text{vol}}\right)$




$$dV_{dipole} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{\vec{P}(\vec{r}')\,d\tau'\cdot\vec{\mathscr{r}}}{|\vec{\mathscr{r}}|^3} = \frac{1}{4\pi\epsilon_0}\frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}d\tau'$$




$$dV = \frac{1}{4\pi\epsilon_0} \frac{\vec{P}(\vec{r}')\cdot(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d\tau'$$

Note

$$\vec{\nabla}' \frac{1}{|\vec{r} - \vec{r}'|} = \left\{ \hat{x}\frac{\partial}{\partial x'}\frac{1}{\sqrt{\phantom{x}}} + \hat{y}\frac{\partial}{\partial y'}\frac{1}{\sqrt{\phantom{x}}} + \hat{z}\frac{\partial}{\partial z'}\frac{1}{\sqrt{\phantom{x}}} \right\}$$

$(x-x')^2 + (y-y')^2 + (z-z')^2$

$$= \hat{x}\frac{-2(x-x')\frac{1}{2}}{(\sqrt{\phantom{x}})^{3/2}} + \hat{y}\frac{-2(y-y')\frac{1}{2}}{(\sqrt{\phantom{x}})^{3/2}} + \hat{z}\frac{-2(z-z')\frac{1}{2}}{(\sqrt{\phantom{x}})^{3/2}}$$

$$= \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$ this looks like

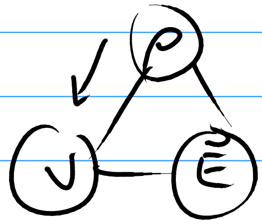
$$dV = \frac{1}{4\pi\epsilon_0} \frac{\vec{P}(\vec{r}')\cdot(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} d\tau' = \frac{1}{4\pi\epsilon_0} \vec{P}(\vec{r}')\cdot\vec{\nabla}'\frac{1}{|\vec{r} - \vec{r}'|} d\tau'$$

$$\vec{\nabla}\cdot(f\vec{A}) = f\,\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}f$$

$$V = \frac{1}{4\pi\epsilon_0}\int \vec{\nabla}'\cdot\left(\frac{\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right)d\tau' - \frac{1}{4\pi\epsilon_0}\int \frac{\vec{\nabla}'\cdot\vec{P}}{|\vec{r}-\vec{r}'|}d\tau'$$

divergence theorem

$$V = \frac{1}{4\pi\epsilon_0}\oint \frac{\vec{P}(\vec{r}')\cdot d\vec{a}'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi\epsilon_0}\int \frac{-\vec{\nabla}'\cdot\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|}d\tau'$$




$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')\,d\tau'}{r} + \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, da'}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\oint \frac{\sigma_b\, da'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi\epsilon_0}\int \frac{\rho_b\, d\tau'}{|\vec{r}-\vec{r}'|}$$

$$\sigma_b = \vec{P}\cdot\hat{n}$$

$$\rho_b = -\vec{\nabla}'\cdot\vec{P}(\vec{r}')$$