Lecture 18

Note Title 2/27/2006

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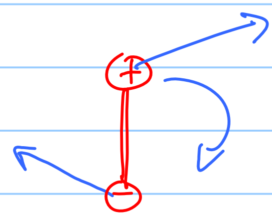


$\vec{F}_{net}$ is up

torque twists $\vec{r} \times \vec{F}$

Net force

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q\left(\vec{E}_+ - \vec{E}_-\right) \rightarrow q\, d\vec{E} \quad \text{small dipole}$$

diff in E at + & −

$$\vec{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$$

$$dE_x(x,y,z) = \frac{\partial E_x}{\partial x} dx + \frac{\partial E_x}{\partial y} dy + \frac{\partial E_x}{\partial z} dz$$

$$= \underbrace{\left(\frac{\partial E_x}{\partial x}\hat{x} + \frac{\partial E_x}{\partial y}\hat{y} + \frac{\partial E_x}{\partial z}\hat{z}\right)}_{\vec{\nabla} E_x} \cdot \underbrace{\left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)}_{d\vec{l}}$$

$d\vec{l}$ ← vector between 2 charges

Also similar thing for E_y & E_z

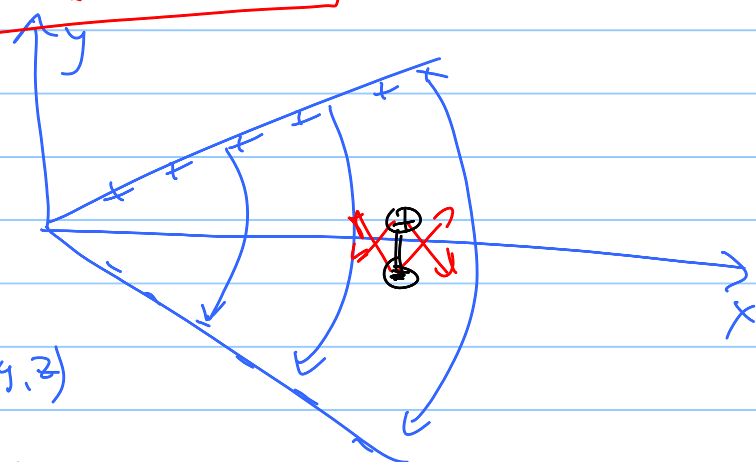
$$d\vec{E} = \underbrace{(d\vec{l}\cdot\vec{\nabla})}_{\text{scalar}}\vec{E} \qquad d\vec{F} = q\,d\vec{E}$$

q

$$q\,dl \to q\,\Delta\vec{l} = \vec{p}$$

$$\boxed{\vec{F}_{\text{on dipole}} = (\vec{p}\cdot\vec{\nabla})\vec{E}}$$

Ex:



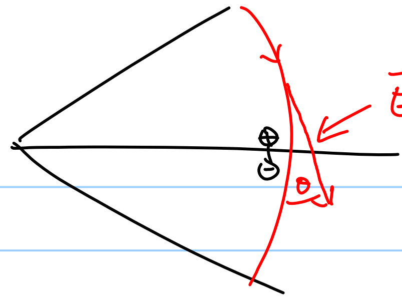
given $\vec{E}(x,y,z)$

$$\vec{p} = P_0\,\hat{y} \quad \text{given}$$

$$\vec{F} = \left(P_0\hat{y}\cdot\left[\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right]\right)\vec{E}$$

$\hat{y}\cdot\hat{x} = 0 \qquad \hat{y}\cdot\hat{z} = 0$

$$\vec{F} = P_0\frac{\partial}{\partial y}\vec{E} = \underline{\boxed{P_0\frac{\partial}{\partial y}E_x\,\hat{x}} + P_0\frac{\partial}{\partial y}E_y}_{\approx 0}\,\hat{y} + \cancel{P_0\frac{\partial}{\partial y}}E_z\,\hat{z} \quad 0$$



$$\vec{E} = -E_0 \underbrace{\cos\vartheta}_{E_y}\,\hat{y} + E_0 \underbrace{\sin\vartheta}_{E_x}\,\hat{x}$$

Say ϑ is small

$$\cos\vartheta \simeq 1 - \frac{\vartheta^2}{2} + \dots$$

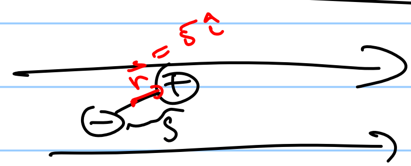
$$\sin\vartheta \simeq \vartheta + \dots$$

$\dfrac{\partial E_x}{\partial y}$ non zero to 1st order in ϑ

$\dfrac{\partial E_y}{\partial y}$ 2nd order in ϑ (for ϑ small $\simeq 0$)

$$\vec{\nabla}(\vec{p}\cdot\vec{E}) \to (\vec{p}\cdot\vec{\nabla})\vec{E}$$

const



$\vec{E}$ uniform

$$\vec{N} = \vec{r}\times\vec{F}_+ + (\underset{\vec{s}/2}{-\vec{r}})\times\vec{F}_-$$

Torque about center mass.