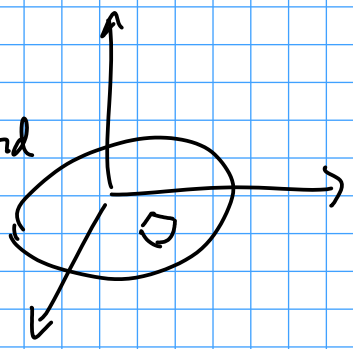
Review: $\vec{F} = q\vec{v}\times\vec{B}$ $\quad d\vec{F} = dq\,\vec{v}\times\vec{B}$

Sheet of charge $\quad dq = \sigma\,da \quad d\vec{F} = \sigma\,da\,\vec{v}\times\vec{B} = \vec{K}\times\vec{B}\,da$

$\vec{K}$

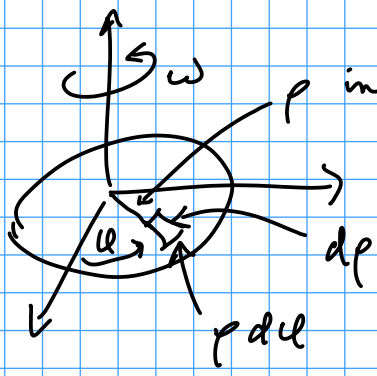
Ex:

Charged record



$$\vec{F} = \int \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ K_x & K_y & K_z \\ B_x & B_y & B_z \end{vmatrix} da$$

Why cartesian coords?



$\hat{\varphi} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$

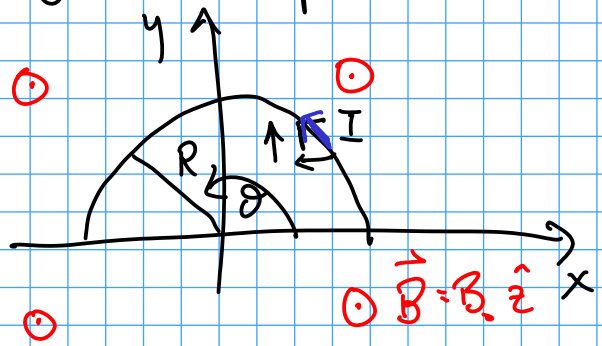
$da = \rho\,d\varphi\,d\rho$

$d\vec{F} = dq\,\vec{v}\times\vec{B} \quad \vec{v} = \rho\omega\hat{\varphi} \quad dq = \sigma\,da$

$$\vec{F} = \int \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \sigma\,da$$

InkSurvey question:

Derive an expression for $d\vec{F}$ on a segment of this wire



$d\vec{\ell} = R\,d\theta\,\hat{\theta} = R\,d\theta\left(-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}\right)$

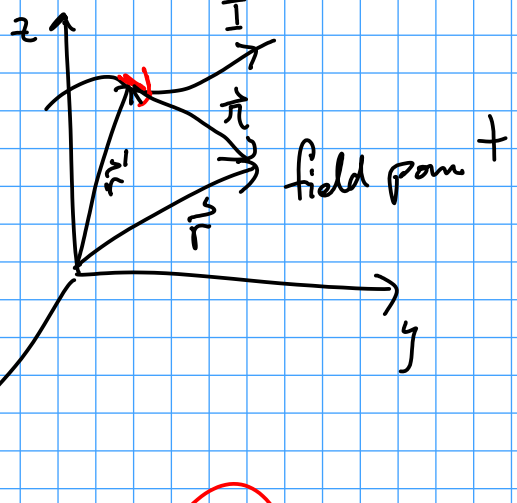
$$d\vec{F} = I\,d\vec{\ell}\times\vec{B} = I\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & B_0 \end{vmatrix} R\,d\theta$$

Find $\vec{B}$ given steady currents (Magnetostatics)

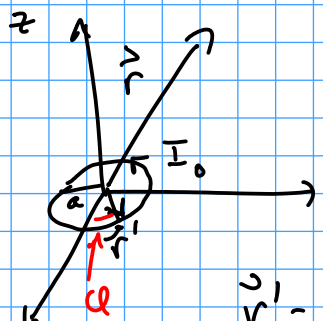
Law of Biot & Savart

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{r}}{r^2}$$ for current in a wire

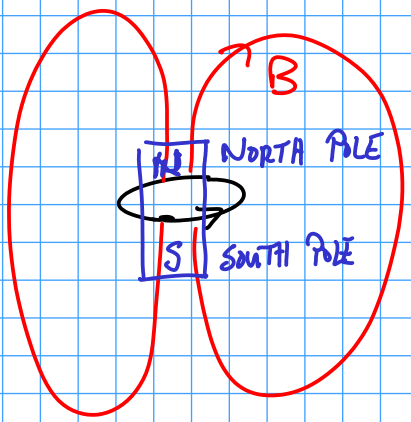
$\vec{B}$ is a function of $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$



Ex: Find $\vec{B}$ from a current loop.



Is this a dipole field?



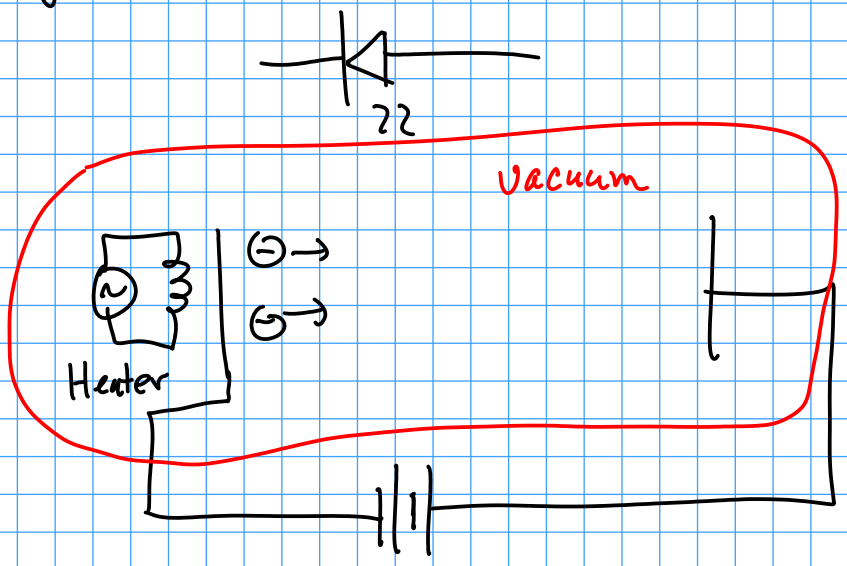
$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{r}' = a\cos\phi'\hat{x} + a\sin\phi'\hat{y} + \phi\hat{z}$

$\vec{r} = \vec{r} - \vec{r}'$ $\quad d\vec{\ell} = a\,d\phi'\hat{\phi}$

Space charge: charge distributed in a volume

Ex: Vacuum diode



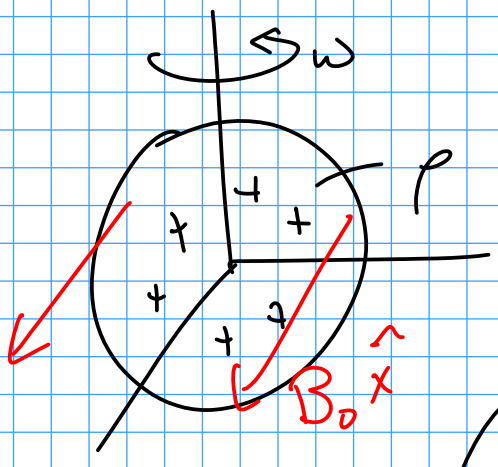
$dq = \rho\, d\tau$

$d\vec{F} = dq\,\vec{v}\times\vec{B} = \rho\, d\tau\,\vec{v}\times\vec{B} = \vec{J}\times\vec{B}\,d\tau$

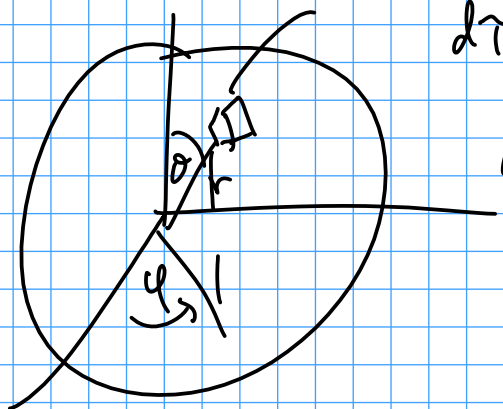
$\vec{J}$ current density $\rho\left(\frac{C}{m^3}\right)\vec{v}\left(\frac{m}{s}\right)$

$\frac{C}{s}\frac{1}{m^2}$ or $\frac{Amps}{m^2}$

Ex:



Find $\vec{F}$ on sphere



$d\tau = r^2 \sin\theta\, d\theta\, d\phi\, dr$

$d\vec{F} = \vec{J}\times\vec{B}\,d\tau$

$\rho\vec{v} = \rho\,\vec{\omega}\times\vec{r}$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \hat{r}}{r^2}$$

For current density $\vec{J}$ what is $\vec{B}$? $I d\vec{\ell} \to \vec{K}\, da \to \vec{J}\, d\tau$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \hat{r}}{r^2} \, d\tau$$