

Total internal reflection + evanescent waves



what are waves + fields in TIR?

$\Rightarrow n_1 > n_2, \theta_i < \theta_c$ refraction away from.



$$\theta_i = \theta_c \quad n_1 \sin \theta_c = n_2 \quad \sin \theta_c = \frac{n_2}{n_1}$$

$$\theta_t = \theta_c$$

At critical angle, wave in region 2 is a plane wave traveling w/ $\vec{E}_2 \parallel$ interface.

- we assume ∞ plane wave input, so at $\theta_i = \theta_c$ wave in region 2 extends to $z \rightarrow \infty$



For $\theta_i > \theta_c$? still TIR, $R = |r|^2 = 1$

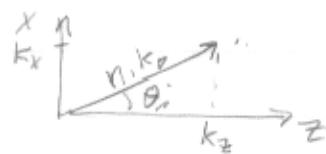
$$\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i > 1 \quad \theta_t \text{ is complex}$$

Instead of using complex θ 's, return to wave vectors:

wave equation yields

$$-\nabla^2 E = n^2 \frac{w^2}{c^2} E \rightarrow \boxed{k_x^2 + k_y^2 + k_z^2 = n^2 k_o^2}$$

for normal plane wave, this is just a vector magnitude:



but \vec{k} can be complex: $\underline{\vec{E} \cdot \vec{E}} = n^2 k_o^2$ not $|\vec{k}|^2 = n^2 k_o^2$

Evanescent wave:

$$\text{region 1: } k_{x_1} = k_0 n_1 \sin \theta_i \quad k_{x_1}^2 + k_{z_1}^2 = k_0^2 n_1^2$$
$$k_{z_1} = k_0 n_1 \cos \theta_i$$

region 2:

$$\text{by phase continuity} \quad k_{x_2} = k_{x_1} = k_0 n_1 \sin \theta_i$$

$$k_{z_2} = \sqrt{n_2^2 k_0^2 - k_{x_2}^2} = \sqrt{n_2^2 k_0^2 - n_1^2 k_0^2 \sin^2 \theta_i}$$
$$= k_0 \sqrt{n_2^2 - n_1^2 \sin^2 \theta_i}$$

always true.

$$\text{for } \theta_i > \theta_c \quad n_1^2 \sin^2 \theta_i > n_2^2$$

$$\rightarrow k_{z_2} = i k_0 \sqrt{n_1^2 \sin^2 \theta_i - n_2^2}$$

evanescent wave is propagating in \hat{x} direction still,
exponentially damped in z :

$$E_t e^{i(k_{x_2}x + k_{z_2}z - \omega t)} = E_t e^{i(k_{x_1}x - \omega t) - |k_{z_2}|z}$$

$$\text{where } E_t = t E_i \quad t = \text{complex}$$

$$E_r = r E_i \quad r = e^{i\phi}$$

Since k_z is pure imaginary, no loss, just reflection
(see homework)

wave picture: vs. z

$$\text{Region 1} \quad E_0 e^{i(k_x x + k_z z)} + E_n e^{i(k_x x - k_z z)}$$

for $\vec{E} \perp$ to POI, $n = n_\perp$

$$n_\perp = \frac{n_1 \cos \theta_0 - n_2 \cos \theta_c}{n_1 \cos \theta_0 + n_2 \cos \theta_c} \quad \begin{aligned} w/n_2 \cos \theta_c &= n_2 \sqrt{1 - \sin^2 \theta_c} \\ &= \sqrt{n_2^2 - n_1^2 \sin^2 \theta_0} \\ &= i\alpha \end{aligned}$$

$$r_+ = \frac{n_1 \cos \theta_0 - i\alpha}{n_1 \cos \theta_0 + i\alpha} = \frac{n^*}{n}$$

$$\text{and } |n_\perp|^2 = \frac{n^*}{n} \cdot \frac{n}{n^*} = 1$$

can write $n_\perp = e^{i\phi}$ with $\phi = \tan^{-1}\left(\frac{-\alpha}{n_1 \cos \theta_0}\right)$

in region 1,

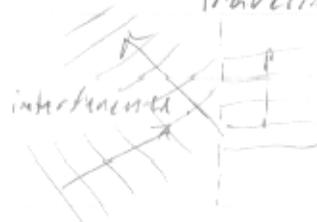
$$E_n = E_0 e^{i k_x x} \left(e^{i k_z z} + e^{-i k_z z} \right) \\ = E_0 e^{i k_x x + \phi/2} \cdot \underbrace{2 \cos(k_z z - \phi/2)}_{\text{standing wave in } z, \text{ w/ phase shift}}$$



$$\text{in region 2} \quad E \sim E_r e^{i k_x x - k_z \alpha z}$$

traveling in x , damped in z

wavefronts,



Reflection from metals

Review boundary conditions

$$\nabla \cdot \vec{D} = 4\pi \rho_s \rightarrow \underline{\epsilon_1 E_1^+ - \epsilon_2 E_2^+ = 4\pi \rho_{sf}}$$

$$\nabla \cdot \vec{B} = 0 \rightarrow \underline{B_1^+ - B_2^+ = 0} \quad \text{free surface charge}$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \rightarrow \underline{E_1^{\parallel} - E_2^{\parallel} = 0}$$

$$\nabla \times \vec{H} = +\frac{1}{c} \frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c} \vec{J} \rightarrow \underline{H_1^{\parallel} - H_2^{\parallel} = \frac{\vec{J}_{sf} \times \hat{n}}{\text{free surface current}}}$$

but if $\vec{J} = \sigma \vec{E}$

a surface current implies a jump in E^{\parallel}

$$\vec{J} = \vec{J}_{sd}(x, y) \delta(z)$$

which violates continuity of E^{\parallel}

∴ no surface currents.

Also ignore surface charge too as before.

∴ B.C. same as in dielectrics, we can use & extend solutions derived earlier w/ complex index

normal incidence

$$E_1^0 = \frac{\tilde{n}_2 - n_1}{\tilde{n}_2 + n_1} E_0^0 \quad \text{reflection}$$

$$E_2^0 = \frac{2n_1}{\tilde{n}_2 + n_1} E_0^0 \quad \text{transmitted}$$

note \tilde{r}, \tilde{t} are complex \rightarrow phase shifts.

limit high σ

$$\tilde{n}_2 = \sqrt{\epsilon_2 + i \frac{4\pi\sigma_2}{\omega}} = \sqrt{i \frac{4\pi\sigma_2}{\omega}}$$

remember $S = \frac{c}{\sqrt{2\pi\sigma\omega}} \approx \text{skin depth}$

$$\sqrt{2\pi\sigma} = \frac{c}{8\sqrt{\omega}}$$

$$\Rightarrow \tilde{n}_2 = \frac{(1+i)}{\sqrt{2}} \sqrt{\frac{4\pi\sigma_2}{\omega}} = \frac{(1+i)c}{8\omega} = \frac{1+i}{k_{\text{mec}} S}$$

now calc \tilde{r}

$$\tilde{r} = \frac{1+i - n_1 k_{\text{mec}} S}{1+i + n_1 k_{\text{mec}} S}$$

\rightarrow sign change by
ord'nt'n

for $S \ll \lambda/n$, \tilde{r} close to 1, $(1+i)^2 \approx 2$

$$\begin{aligned} \tilde{r} &\approx \left(\frac{1+i - n_1 k_{\text{mec}} S}{1+i + n_1 k_{\text{mec}} S} \right)^2 \\ &\approx \frac{1+i - 2n_1 k_{\text{mec}} S}{1+i + 2n_1 k_{\text{mec}} S} \end{aligned} \quad \left. \begin{array}{l} \text{skip} \\ \text{skip} \end{array} \right\}$$

$$|\tilde{r}|^2 = R \approx \frac{1}{2}(1 - 2n_1 k_{\text{mec}} S)^2 + \frac{1}{2} \approx 1 - 2n_1 k_{\text{mec}} S \quad \rightarrow \text{small losses}$$

Oblique incidence onto metal surface,

- continue to use Fresnel equations
- be careful with sign conventions
 - best to just force $r \rightarrow -1$ at $\theta = 0^\circ$ for both r_{\perp} and r_{\parallel}
- variable phase shifts with θ , and polarization
 - reflectivity is always best with "S"

wave inside metal:

use Fresnel eq. to get ampl., phase.

internal wave is then

$$S: E_0^{\perp} t_{\perp} e^{i(k_x x + k_z z - \omega t)}$$

now $k_x x \rightarrow k_m n_s \sin \theta_0 \cdot x$ is still the same. (just as in TIR)

$k_z \rightarrow$ complex (not pure imaginary)

$\text{Re}(k_z) \rightarrow$ osc. period,

∴ loss is only in z direction.

phase fronts are in direction of $\text{Re}(\vec{k})$:

$$k_x x + \text{Re}(k_z) z$$

in general, this is different than Snell's law

Power flow

time avg Pointing vector with complex fields

$$\langle \vec{S} \rangle = \frac{\epsilon}{8\pi} \operatorname{Re}(\vec{E} \times \vec{H}^*) \quad \vec{S} \text{ must be real}$$

put \vec{H} in terms of \vec{E} :

$$\nabla \times \vec{E} = i \vec{k} \times \vec{E} = -\frac{\mu}{c} \frac{d\vec{H}}{dt} = i \frac{\omega}{c} \mu \vec{H}$$

$$\vec{H} = \frac{i \vec{k} \times \vec{E}}{\mu \omega / c}$$

$$\langle \vec{S} \rangle = \frac{\epsilon}{8\pi} \operatorname{Re}(\vec{E} \times (\vec{k}^* \times \vec{E}^*))$$

$$= \frac{c^2}{8\pi \mu \omega} \operatorname{Re}[\vec{k}^*(\vec{E} \cdot \vec{E}^*) - \vec{E}^*(\vec{k}^* \cdot \vec{E})]$$

for S-polarization $\vec{E} \cdot \vec{E}^* = 0$ automatically (E_y, k_x, μ_2)

for P-polarization $\nabla \cdot \vec{E} = i \vec{k} \cdot \vec{E} = 4\pi p \rightarrow 0$ in dielectric

$\rightarrow 0$ for quasi-static.

dropping second term, $\langle \vec{S} \rangle$ goes along $\operatorname{Re}(\vec{E})$,

i.e. oscillating component of wave.

TIR $\langle \vec{S} \rangle$ is along interface

metal $\langle \vec{S} \rangle$ has dominant component along interface.

component along \vec{k} \rightarrow Ohmic loss.