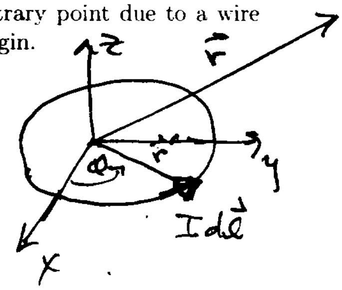
7. Derive an integral expression for the magnetic vector potential at an arbitrary point due to a wire loop carrying current I_0 of radius R in the xy plane and centered at the origin.



$$\boxed{\text{Defn: } \vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\, d\tau}{\mathscr{r}}} \rightarrow \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l}}{\mathscr{r}}$$

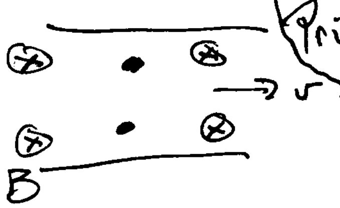
$$\vec{r}' = x'\hat{x} + y'\hat{y} = R\cos\varphi'\,\hat{x} + R\sin\varphi'\,\hat{y}$$

$$d\vec{r}' = d\vec{l} = -R\sin\varphi'\, d\varphi'\,\hat{x} + R\cos\varphi'\, d\varphi'\,\hat{y}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$\vec{r}'$ = above $\quad |\vec{\mathscr{r}}| = \sqrt{(x - R\cos\varphi')^2 + (y - R\sin\varphi')^2}$ $\quad$ limits φ': $0 \to 2\pi$

8. The current in a river (think ions in the water) in northern Canada moves at speed v_0 perpendicular to a bridge while the magnetic field, B_0, points perpendicular to water surface. A wooded suspension bridge over the river is supported from two metal towers in the water which are separated by a distance L. Derive an expression for the voltage between the towers?



Prin: $\boxed{\vec{F} = q\vec{v}\times\vec{B}} = qvB$ so + charges move to one post & − charges to the other. This generates an $\vec{E}$ field until $\boxed{\sum F = qvB + qE} = 0$ $\quad |\vec{E}| = |v\vec{B}|$

$$|\Delta V| = \left|-\int \vec{E}\cdot d\vec{l}\right| = \int_0^L vB\, dy = vBL$$

9. Explain in detail how you would calculate the charge density σ on a spherical metal shell of radius R with voltage V_0 on the upper hemisphere and $-V_0$ on the lower hemisphere. The two hemispheres are electrically insulated from each other.

Principles: Solve problem where there is no charge & apply bndry conditions to accnt for charge. $\boxed{\nabla^2 V = 0}$

in spherical words use Separation of varibles

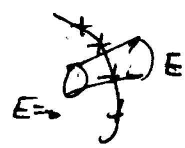
$V(r,\theta) = R(r)\,\Theta(\theta)$. For arbitrary bndry conditions we must sum these solns $V(r,\theta) = \sum a_n R(r)\,\Theta(\theta)$

coeff determined by bndry voltages

use Fouriers trick to find a_n.

Knowing V near bndry we can find $\vec{E} = -\vec{\nabla} V \rightarrow -\frac{\partial V}{\partial n}$

Near bndry Gauss law relates σ to E $\quad \oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$



$$E_\perp^{above} - E_\perp^{below} = \frac{\sigma}{\epsilon_0}$$