Recitation 2 – Dielectric interfaces

a) Let's suppose we have TM-polarized plane waves incident on the boundary between two dielectrics with indices n_1 and n_2. The incident light makes some nonzero angle θ with the optical axis. Sketch that situation, including the directions of the *k* vector, E-field, and B-field for each of the three waves involved. Label your axes.

b) Given an incident electric field of the form

$$\vec{E}_I(\vec{x},t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t+\delta)}$$

What phase angle δ could we choose to represent an incident E-field that has zero magnitude when $\vec{x}$ and t are zero?

c) Write the four field boundary conditions that we can most easily apply to dielectric interface problems. Keep them in a general form; don't adapt them to this specific problem yet.

d) Explain why we don't directly use the boundary condition that reads $E_{1,\perp} - E_{2,\perp} = {}^{\sigma}/_{\varepsilon_0}$.

e) Explain why we can be sure that the incident, reflected, and transmitted fields will all have the same frequency ω. Be specific. Also tell me whether there's one particular boundary condition that contains this information, or if more than one has it.

f) How, specifically, is the wavelength in medium 2 related to the wavelength in medium 1, and how do you know?

g) Write full equations for the incident, reflected, and transmitted E-fields. Then use each boundary condition to generate an equation that relates the amplitudes of those fields. Get things simplified to the point that there's nothing left to do but solve for the field amplitude ratios, and write things in terms of the given indices of refraction. You can set the arbitrary phase angle δ to zero for simplicity.

h) If you have time, go ahead and solve for the ratios $\frac{E_0'}{E_0}$ and $\frac{E_0''}{E_0}$ to obtain two of the Fresnel equations.