MACS 332A - Fall 2008

NAME:____________________________________ **Exam I**

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. Please enclose your final answers in boxes.

1. (12 points) Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 0 \\ 2 & -1 & 1 \end{bmatrix}$. A has distinct integer eigenvalues. Find the eigenvalues of A.

2. Let $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$. $\lambda = 1$ is an eigenvalue for A and has a multiplicity of 3.

   (a) (10 points) Find a basis for the eigenspace corresponding to $\lambda = 1$.

   (b) (6 points) If A is diagonalizable, find P and D that diagonalizes A. If A is not diagonalizable, explain why.

3. (10 points) Use coordinate vectors to determine whether $H = \{2x, x^3 - 3, x - 4x^3, x^3 + 18x - 9\}$ forms a basis for $\mathbb{P}_3$. Justify your answer.

4. (12 points) The set of all 4×4matrics, $M_{4\times4}$, is a vector space. Prove whether or not that $H = \{D \in M_{4\times4} \,:\, D \text{ is diagonal } \}$ is a subspace of $M_{4\times4}$.

5. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 4 & 6 \\ 3 & 3 & 6 \end{bmatrix}$. Find

   (a) (10 points) Nul$A$

   (b) (5 points) Col$A$

   (c) (3 points) dim Row$A =$

   (d) (3 points) rank$A =$

6. Find a basis in $\mathbb{R}^3$ for the set of vectors in the plane $2x_1 - 3x_2 + 6x_3 = 0$.

7. (12 points) Let $A = \begin{bmatrix} 5 & -1 \\ 5 & 1 \end{bmatrix}$. Find P and C where C has the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ and such that $A = PCP^{-1}$.

8. (12 points) The President of the United States tells person A his intention to run or not to run in the next election. Then A relays the news to B, who in turn relays the message to C, and so forth, always to some new person. We assume that there is a probability of 0.4 that a person will change the answer from yes to no when transmitting it to the next person and a probability of 0.3 that he or she will change it from no to yes.

   (a) Set up a stochastic matrix for this problem.

   (b) If the president initially says yes he will run, what is the probability that C will hear yes from B?