MATH332-Linear Algebra **Homework One**

The Row Reduction Algorithm

Text: 1.1-1.2 Section Overviews: 1.1-1.2

Quote of Homework One	
It is absolutely essential that you do as many problems as you can: mathematics is not a spectator sport!	
	Dr. Paul Martin : Professor of Mathematics at CSM

1. Reading

Read the following:

- Textbook : [1.2.14-20] Chapter 1, Section 2, Pages 14-20
- Wikipeida : Article on Gaussian Elimination, `http://en.wikipedia.org/wiki/Gaussian_elimination`

The first should be read for comprehension while the second should be read for breadth. That is, you should be trying comprehend the textbook material material deeply. However, the wikipedia article contains many different topics and jargon, which obscures the point. Read as much of the article that time and your background allows. Do not spend more than twenty minutes reading wikipedia.

2. Writing

Making sure to use the following vocabulary words,

- Pivot
- Echelon Form
- Elementary Row Operations

explain the row-reduction algorithm.

3. Arithmetic

Using the row-reduction algorithm find the **reduced** row-echelon form for each of the following matrices. Be sure to check your work with some computational tool.[1] The following are some recommendations:

- TI-83 and others :
- SAGE : `http://www.sage.org`
- Online Linear Algebra Toolkit : `http://www.math.odu.edu/~bogacki/cgi-bin/lat.cgi` [2]

$$\mathbf{A}_1 = \begin{bmatrix} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 6 & 18 & -4 & 20 \\ -1 & -3 & 8 & 4 \\ 5 & 15 & -9 & 11 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 3 & 0 \end{bmatrix}, \mathbf{A}_4 = \begin{bmatrix} 1 & 2 & 3 & 10 \\ 2 & 4 & 6 & 20 \\ 3 & 6 & 9 & 30 \end{bmatrix}, \mathbf{A}_5 = \begin{bmatrix} 5 & 3 & 22 \\ -4 & 7 & 20 \\ 9 & -2 & 15 \end{bmatrix},$$

$$\mathbf{A}_6 = \begin{bmatrix} 1 & -5 & 1 \\ -1 & 7 & 1 \\ -3 & 8 & h \end{bmatrix}, \mathbf{A}_7 = \begin{bmatrix} 1 & -3 & 5 \\ -3 & 9 & -7 \\ 2 & -6 & h \end{bmatrix}, \mathbf{A}_8 = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{bmatrix}, \mathbf{A}_9 = \begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix}, \mathbf{A}_{10} = \begin{bmatrix} 3 & 6 & 7 \\ 0 & 2 & 1 \\ 2 & 3 & 4 \end{bmatrix},$$

$$\mathbf{A}_{11} = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}.$$

[1] For $\mathbf{A}_6$ and $\mathbf{A}_7$, h is a parameter. You can check your work by choosing a value for h at the final step and comparing to a computational device.

[2] This one is interesting since it shows the changes to the matrix at each step.

4. Analysis

The row-reduction algorithm and the associated reduced row-echelon form plays an interesting role in linear algebra. While the original information is present, this simplified form tell us everything we would like to know. Right now, we have only numbers. To gather the meaning of these numbers we must make the logical connection,

$$\begin{array}{cccccc} a_{11}x_1+ & a_{12}x_2+ & a_{13}x_3+ & \cdots & +a_{1n}x_n & = b_1, \\ a_{21}x_1+ & a_{22}x_2+ & a_{23}x_3+ & \cdots & +a_{2n}x_n & = b_2, \\ a_{31}x_1+ & a_{32}x_2+ & a_{33}x_3+ & \cdots & +a_{3n}x_n & = b_3, \\ \vdots & \vdots & & \ddots & \vdots & \vdots \\ a_{m1}x_1+ & a_{m2}x_2+ & a_{m3}x_3+ & \cdots & +a_{mn}x_n & = b_m, \end{array} \iff \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & \ddots & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_m \end{bmatrix}, \tag{1}$$

which we condense into the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$, where $\mathbf{A}$ is a matrix containing the coefficients, $\mathbf{b}$ is a vector of intercept points and $\mathbf{x}$ is a vector of unknowns. Suppose we are working with a linear system by $\mathbf{A}_i$ what is the solution to $\mathbf{A}_i\mathbf{x} = \mathbf{0}$? Check you work with a couple of systems by showing that the solution you find from the reduced row-echelon form satisfies both the original linear system and its reduced form.

5. Reflection

Thinking back to the start of this assignment, what did you learn? What should you make sure to remember for the future? What questions remain?