

Simple examples from plasma physics

- Debye shielding, cyclotron frequency, plasma freq.

Free electron gas model:

- no collisions \rightarrow no resistance
- neutralizing background of ions $\rho_i = \rho_e$
- charge density $\rho_e = -e N_e$ $N_e = \#/\text{vol}$
 cm^{-3} (lab units)

Debye shielding

charges arrange to cancel E-field in plasma

- "cold" plasma: +ve test charge accumulates -ve charge
 $\textcircled{+} \rightarrow$ neutralize.

- "warm" plasma:

thermal pressure balances ES force.

Boltzmann distrib: $-(\frac{1}{2}mv^2)/kT$

$$f(x, v) = A e^{-\frac{1}{2}mv^2/kT} \quad \begin{matrix} \text{no potential} \\ \text{constant density, } T \end{matrix}$$

normalize:

$$n(x) = \int f(x, v) dv \rightarrow A = n_0 \left(\frac{m}{2\pi kT} \right)^{1/2}$$

Now add potential $U(x)$

$$f(x, v) = A \exp \left(- \left(\frac{1}{2}mv^2 + U(x) \right) / kT \right)$$

$$= A \exp \left(- \frac{U(x)}{kT} \right)$$

$$n(x) = n_{\text{ref}} e^{-U(x)/kT}$$

$$n_{\text{ref}} = N_e \text{ where } U=0$$

example - for gravity $U = mgx$
 → exponential fall off in density w/ altitude.

electric pot'l $U = q\phi = -e\phi$

pick ref at $x = \infty$ (far from test charge)

$$n(x=\infty) = n_{\infty}$$

$$\rightarrow n_e(x) = n_{\infty} e^{e\phi/kT_e}$$

consider a grid at a fixed pot'l (one-dim)



solve for $\phi(x) \rightarrow n_e(x)$

$$\nabla \cdot \vec{E} = 4\pi\rho = -4\pi e(n_e - n_i)$$

$$\vec{E} = -\nabla\phi$$

$$\rightarrow \text{poisson eqn } \nabla^2\phi = 4\pi e(n_e - n_i)$$

assume long and Axial:

$$1 \rightarrow \frac{d^2\phi}{dx^2} = 4\pi e n_{\infty} (e^{e\phi/kT_e} - 1)$$

where $e\phi/kT_e \ll 1$, expand expn.

$$e^x \approx 1 + x + \frac{1}{2}x^2 \sim$$

keep lowest order term

$$\frac{d^2\phi}{dx^2} = \underbrace{\frac{4\pi n_{\infty}}{kT_e}}_{} \phi$$

$$\text{define } \lambda_D^{-1} = \left(\frac{4\pi n_{\infty} e^2}{kT_e} \right)^{1/2} \quad \text{dimensions } m^{-2}$$

n_e = background density

$$\rightarrow \frac{d^2\phi}{dx^2} = \lambda_D^{-2} \phi \quad + \text{sign} \rightarrow \text{exponential solutions}$$

$$\phi = \phi_0 e^{\pm x/\lambda_D} = \phi_0 e^{-|x|/\lambda_D}$$

calculate λ_D = Debye length

$$\text{gaussian } \frac{4\pi e^2}{4\pi\epsilon_0} \rightarrow \text{SI } e^2/\epsilon_0$$

$$\text{or } e^2 \sim \frac{1}{4\pi\epsilon_0} e^2$$

$$\lambda_D = \left(\frac{\epsilon_0 k_B T_e}{n_e e^2} \right)^{1/2}$$

dimension check: $k_B T_e \sim \text{energy}$

$e^2/r \sim \text{energy} \rightarrow e^2 \sim \text{energy/len}$

$$\lambda_D \sim \left(\frac{\text{energy}}{(\text{len})^3 \text{energy} \cdot \text{len}} \right)^{1/2} \sim \text{len.}$$

experimental units:

$$k_B T_e \sim \text{eV}$$

$$n_e \sim \text{cm}^{-3}$$

$$\rightarrow \lambda_D = 743 \left(\frac{k_B T_e}{n_e} \right)^{1/2} \text{cm} \quad k_B T_e \text{ in eV} \\ n_e \text{ in cm}^{-3}$$

at 1 Torr (760 Torr at STP), room temp

$$\text{ideal gas } PV = N k_B T \rightarrow n = P/k_B T$$

$$k_B T \sim 1/40 \text{ eV}$$

$$n \sim 3 \times 10^{16} \text{ cm}^{-3}$$

full ionization, $k_B T_e = 1 \text{ eV}$

$$\rightarrow \lambda_{D_e} \sim 40 \text{ nm}$$

calc. # particles in sphere of radius λ_D :

$$N_D = n_e \cdot \frac{4}{3} \pi \lambda_D^3 \sim 10$$

if $N_D \gg 1 \rightarrow \text{collective behavior}$

Cyclotron frequency:

moving electron with $\vec{V}_e \perp \vec{B}$ has circular path

$$\vec{F} = q\vec{E} + q\frac{\vec{V}}{c} \times \vec{B} \quad \text{gaussian}$$

HW: solve for eqn.

$$\text{scaling} \quad F = m \frac{dv}{dt} \sim \frac{ev}{c} B_0$$

$$\rightarrow \text{frequency} \sim \frac{eB_0}{m_e c} \equiv \omega_c$$

go to eqn to figure units:

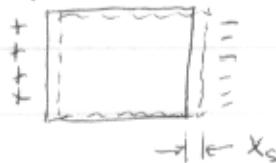
$$\text{SI} \quad F \sim -e\vec{v} \times \vec{B}$$

\therefore take B_0/c (gaussian) $\rightarrow B_0$ (SI)

then use B_0 in tesla, e in coulombs.

Plasma frequency

consider slab of plasma, with all electrons displaced by x_s



net $\vec{E} = 0$ outside, $E_{\text{inside}} \propto x_s$ (use Gaussian)

\rightarrow spring like restoring force

electrons collectively oscillate at

$$\omega_p = \left(\frac{4\pi n e^2}{m_e} \right)^{1/2} \equiv \text{plasma frequency}$$

5.7×10^4

$$\text{SI} \quad \omega_p = \left(\frac{n e^2}{\epsilon_0 m_e} \right)^{1/2} = \text{ne}^{1/2} \text{ rad/s} \quad n_e \text{ in cm}^{-3}$$

$$\text{at 1 Torr} \quad \omega_p \sim 10^{13} \text{ rad/s}, \quad \gamma = \frac{\omega_p}{\pi} \approx 1 \text{ THz}$$