Reading Today: 11.2

Monday: Review

Radiation from point charges

and wierdness

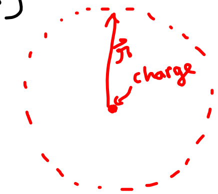
$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left(\underbrace{(c^2-v^2)\vec{u}}_{\text{velocity field}} + \underbrace{\vec{\mathscr{r}}\times(\vec{u}\times\vec{a})}_{\text{acceleration field}} \right)$$

$$\vec{u} = c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\hat{\mathscr{r}}\times\vec{E} \quad ; \quad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$\Rightarrow \vec{S} = \frac{1}{\mu_0}\vec{E}\times\left(\frac{1}{c}\hat{\mathscr{r}}\times\vec{E}\right)$$

$$= \frac{1}{\mu_0 c}\hat{\mathscr{r}}E^2 - \vec{E}(\hat{\mathscr{r}}\cdot\vec{E})$$



$(\text{Velocity field term})^2 \propto \frac{1}{\mathscr{r}^4}$

$(\text{vel. field}\cdot\text{accel. field}) \propto \frac{1}{\mathscr{r}^3}$

$(\text{accel. field})^2 \propto \frac{1}{\mathscr{r}^2}$ $\qquad \lim_{\mathscr{r}\to\infty}$

Surface area of our sphere $\propto \mathscr{r}^2 = 4\pi\mathscr{r}^2$

In $\lim \mathscr{r}\to\infty$, first two terms don't contribute.

I just look at $\vec{E}_{rad} = \frac{q}{4\pi\epsilon_0}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left(\vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right)$

What we're really concerned with is

$\vec{S}\cdot d\vec{a} = \vec{S}\cdot\mathscr{r}^2\sin\theta\, d\theta\, d\phi\, \hat{\mathscr{r}}$

$\vec{E}_{rad} \perp \hat{\mathscr{r}} \Rightarrow \underbrace{-\vec{E}(\hat{\mathscr{r}}\cdot\vec{E})}_{\text{this portion of } \vec{S}}$ doesn't contribute.

$$\vec{S}\cdot d\vec{a} = \frac{1}{\mu_0 c}E_{rad}^2\,\mathscr{r}^2\,d\Omega \leftarrow \sin\theta\, d\theta\, d\phi$$

$$\vec{S}\cdot d\vec{a} = \frac{1}{\mu_0 c} E_{rad}^2 \mathscr{r}^2 \, d\Omega \Leftarrow \sin\theta \, d\theta \, d\phi$$

$$\vec{E}_{rad} = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3} \underbrace{\left(\vec{\mathscr{r}} \times (\vec{u}\times\vec{a})\right)}_{\vec{u}(\vec{\mathscr{r}}\cdot\vec{a}) - \vec{a}(\vec{\mathscr{r}}\cdot\vec{u})}$$

$c\mathscr{r}$

Pick a refence frame wher $\vec{v} = \emptyset \rightarrow \vec{u} = c\hat{\mathscr{r}}$

at t_r

$$c\hat{\mathscr{r}}(\vec{\mathscr{r}}\cdot\vec{a}) - \vec{a}c\mathscr{r}$$

$$\rightarrow \vec{E}_{rad} = \frac{q}{4\pi\epsilon_0} \frac{\mathscr{r}}{c^3 \mathscr{r}^3} \cdot c\mathscr{r} \left(\hat{\mathscr{r}}(\hat{\mathscr{r}}\cdot\vec{a}) - \vec{a}\right)$$

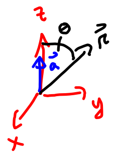
$$= \frac{q\mu_0 \cancel{c^2}}{4\pi} \frac{1}{\cancel{c}\mathscr{r}} \left(\hat{\mathscr{r}}(\hat{\mathscr{r}}\cdot\vec{a}) - \vec{a}\right)$$

$$= \frac{\mu_0 q}{4\pi \mathscr{r}} \left(\hat{\mathscr{r}}(\hat{\mathscr{r}}\cdot\vec{a}) - \vec{a}\right)$$

$$\vec{S}\cdot d\vec{a} = \frac{1}{\mu_0 c}\left(\frac{\mu_0 q}{4\pi \mathscr{r}}\right)^2 \left[(\hat{\mathscr{r}}\cdot\vec{a})^2 + a^2 - 2(\hat{\mathscr{r}}\cdot\vec{a})^2\right] \cdot \mathscr{r}^2 d\Omega$$

$$= \frac{\mu_0 q^2}{16\pi^2 \cancel{\mathscr{r}^2} c}\left(a^2 - (\hat{\mathscr{r}}\cdot\vec{a})^2\right) \cancel{\mathscr{r}^2} \, d\Omega$$

$$= \frac{\mu_0 q^2}{16\pi^2 c}\left(a^2 - (\hat{\mathscr{r}}\cdot\vec{a})^2\right) \sin\theta \, d\theta \, d\phi$$



$$= \frac{\mu_0 q^2}{16\pi^2 c}\left(a^2 - a^2\cos^2\theta\right) \sin\theta \, d\theta \, d\phi$$

$$= \frac{\mu_0 q^2}{16\pi^2 c} a^2 \sin^3\theta \, d\theta \, d\phi$$

Power radiated: $P = \int \vec{S}\cdot d\vec{a} = \int_0^{2\pi}\int_0^{\pi} \frac{\mu_0 q^2 a^2}{16\pi^2 c} \sin^3\theta \, d\theta \, d\phi$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$$

Larmor Formula. $\rightarrow P = \dfrac{\mu_0 q^2 a^2}{6\pi c}$

You can solve for P when $\vec{v} \neq \emptyset$

$$\frac{dP}{d\Omega} = \left(\frac{\vec{\mathscr{r}} \cdot \vec{u}}{\mathscr{r} c}\right) \frac{1}{\mu_0 c} E_{rad}^2 \mathscr{r}^2$$

$$= \frac{q^2}{16\pi^2 \epsilon_0} \frac{|\hat{\mathscr{r}} \times (\vec{u} \times \vec{a})|^2}{(\hat{\mathscr{r}} \cdot \vec{u})^5}$$

$$P = \frac{\mu_0 q^2 \gamma^6}{6\pi c}\left(a^2 - \left|\frac{\vec{v} \times \vec{a}}{c}\right|^2\right)$$

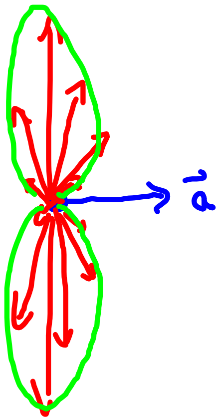
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

As you approach $v = c$ P goes through the roof $\propto \gamma^6$ if $\vec{v} \parallel \vec{a}$, γ^4 if $\vec{v} \perp \vec{a}$.

Direction

Stationary guy

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2}{16\pi^2} a^2 \sin^2\theta$$



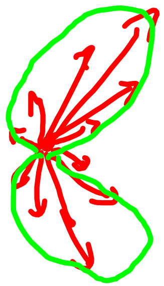
Guy in motion.

$$\frac{dP}{d\Omega} = \left(\frac{\vec{\mathscr{r}}\cdot\vec{u}}{\mathscr{r}c}\right)\frac{1}{\mu_0 c}E_{rad}^2\,\mathscr{r}^2$$

$\vec{a}, \vec{v}$ are $\parallel$

$$\frac{dP}{d\Omega}\bigg|_{\vec{a},\vec{v}\,\parallel} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\,\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

β = v/c



Radiation Reaction

$$P_{rad} = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$\vec{F}_{rad} \cdot \vec{v} = -\frac{\mu_0 q^2 a^2}{6\pi c}$$

this new force that's stealing energy from the charge.

This doesn't work because there are more fields than just E_{rad} (velocity fields)

Ok, to deal with the fact that $\vec{v}$ fields can contribute, you pick a situation where they don't : $E_{vel.\ fields}\big|_{t=t_{init}} = E_{vel.\ fields}\big|_{t=t_{final}}$

Periodic motion:

$t_2 - t_1 = T$ period

$$\int_{t_1}^{t_2} \vec{F}_{rad} \cdot \vec{v}\, dt = -\frac{\mu_0 q^2}{6\pi c} \int_{t_1}^{t_2} a^2\, dt$$

$$\int_{t_1}^{t_2} a^2\, dt = \int_{t_1}^{t_2} \frac{d\vec{v}}{dt} \cdot \frac{d\vec{v}}{dt}\, dt$$

$u = \frac{d\vec{v}}{dt} \rightarrow du = \frac{d^2\vec{v}}{dt^2} dt$

$dv = \frac{d\vec{v}}{dt} dt \rightarrow v = \vec{v}$

$$= \cancel{\left(\vec{v} \cdot \frac{d\vec{v}}{dt}\right)\Big|_{t_1}^{t_2}}^{\,\emptyset} - \int_{t_1}^{t_2} \vec{v} \cdot \frac{d^2\vec{v}}{dt^2}\, dt$$

$$\int_{t_1}^{t_2} \underbrace{\left(\vec{F}_{rad} + \xi \frac{d\vec{a}}{dt}\right)}_{\text{If this } \emptyset \text{, then above is}} \cdot \vec{v}\, dt \qquad \xi = -\frac{\mu_0 q^2}{6\pi c}$$

$$\vec{F}_{rad} = \frac{\mu_0 q^2 \dot{\vec{a}}}{6\pi c}$$

$$\vec{F}_{rad} = \frac{\mu_0 q^2 \dot{\vec{a}}}{6\pi c}$$

Say no external force

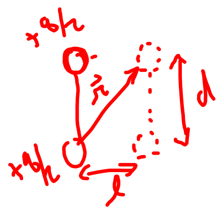
$$\Sigma \vec{F} = \emptyset = m\vec{a} - \frac{\mu_0 q^2 \dot{\vec{a}}}{6\pi c}$$

$$\frac{\mu_0 q^2 \dot{\vec{a}}}{6\pi c} = m\vec{a}$$

$$\frac{\mu_0 q^2}{6\pi c m} \frac{d\vec{a}}{\vec{a}} = dt$$

$$\vec{a} = \vec{a}_0 e^{+\eta t} \qquad \eta = \frac{6\pi c m}{\mu_0 q^2}$$

Another look at $\vec{F}_{self}$.



$$\vec{F}_{self} = \frac{q}{2}\left(\vec{E}_1 + \vec{E}_2\right)$$

$$= \frac{q^2}{8\pi\epsilon_0 c^2} \frac{(\ell c^2 - a d^2)}{(\ell^2 + d^2)^{3/2}} \hat{x}$$

$$\vec{F}_{self} = \frac{q^2}{4\pi\epsilon_0}\left[\frac{-\vec{a}}{4c^2 d} + \frac{\dot{\vec{a}}}{3c^3} + \mathcal{O}(d)\right]\hat{x}$$

ignore him for a sec.

$$\sum \vec{F}_{ext} = m\vec{a} - \vec{F}_{self}$$

$$= m\vec{a} + \left(\frac{q^2}{16\pi\epsilon_0 c^2 d}\right)\vec{a}$$

$$\sum \vec{F}_{ext} = \left(m + \frac{q^2}{16\pi\epsilon_0 c^2 d}\right)\vec{a} \qquad \frac{q^2}{16\pi\epsilon_0 c^2 d} = m_0$$

Ok, except as $d \to 0$ it goes to infinity.

The $\dot{a}$ term gives back ½ Abraham-Lorentz formula. The other half comes from including self, self force {force of each end on themself}.