

**Problem 3.13**

$$V(x, y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a) \quad (\text{Eq. 3.36}); \quad \sigma = -\epsilon_0 \frac{\partial V}{\partial n} \quad (\text{Eq. 2.49}).$$

So

$$\begin{aligned} \sigma(y) &= -\epsilon_0 \frac{\partial}{\partial x} \left\{ \frac{4V_0}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a) \right\} \Big|_{x=0} = -\epsilon_0 \frac{4V_0}{\pi} \sum_{n=1,3,5,\dots} \frac{1}{n} \left( -\frac{n\pi}{a} \right) e^{-n\pi x/a} \sin(n\pi y/a) \Big|_{x=0} \\ &= \boxed{\frac{4\epsilon_0 V_0}{a} \sum_{n=1,3,5,\dots} \sin(n\pi y/a)}. \end{aligned}$$

Or, using the closed form 3.37:

$$\begin{aligned} V(x, y) &= \frac{2V_0}{\pi} \tan^{-1} \left( \frac{\sin(\pi y/a)}{\sinh(\pi x/a)} \right) \Rightarrow \sigma = -\epsilon_0 \frac{2V_0}{\pi} \frac{1}{1 + \frac{\sin^2(\pi y/a)}{\sinh^2(\pi x/a)}} \left( \frac{-\sin(\pi y/a)}{\sinh^2(\pi x/a)} \right) \frac{\pi}{a} \cosh(\pi x/a) \Big|_{x=0} \\ &= \frac{2\epsilon_0 V_0}{a} \frac{\sin(\pi y/a) \cosh(\pi x/a)}{\sin^2(\pi y/a) + \sinh^2(\pi x/a)} \Big|_{x=0} = \boxed{\frac{2\epsilon_0 V_0}{a} \frac{1}{\sin(\pi y/a)}}. \end{aligned}$$