

Radiation from accelerated charges \rightarrow [Larmor $P_{\text{radi}} = \frac{2}{3} \frac{e^2 a^2}{c^3}$]
 - at far distance from source, only radiation terms matter.

$$\vec{E}_a = \left. \frac{e}{c^2 K^3 R^3} \vec{R} \times ((\vec{R} - \vec{\beta} R) \times \vec{a}) \right|_{t_0}$$

for most part, we're interested in calculating radiated power
 - leave it understood that the emitted power is evaluated at t_0 .
 - suppress $[]$ notation.

Even at low velocities, $\beta \ll 1$, accel \rightarrow radiation.

let $\beta \rightarrow 0$ $K \rightarrow 1$

$$\vec{E}_a = \frac{e}{c^2 R^3} \vec{R} \times (\vec{R} \times \vec{a}) = \frac{e}{c^2 R^3} (\vec{R}(\vec{R} \cdot \vec{a}) - \vec{a} R^2) \quad \text{note } \vec{E}_a \perp \vec{R} \text{ transverse}$$

$$\vec{B}_a = \frac{\vec{R} \times \vec{E}_a}{R}$$

$$\vec{S}_a = \frac{c}{4\pi} \vec{E}_a \times \vec{B}_a = \frac{c}{4\pi} \vec{E}_a \times \left(\frac{\vec{R} \times \vec{E}_a}{R} \right) = \frac{c}{4\pi R} \left(\vec{R} E_a^2 - \vec{E}_a (\vec{E}_a \cdot \vec{R}) \right)$$

$$\therefore \vec{S}_a = \frac{c}{4\pi} E_a^2 \vec{R} \quad \text{in direction outward from retarded position}$$

evaluate:

$$\begin{aligned} E_a^2 &= \frac{e^2}{c^4 R^6} (\vec{R}(\vec{R} \cdot \vec{a}) - \vec{a} R^2)^2 = \frac{e^2}{c^4 R^6} (R^2 (\vec{R} \cdot \vec{a})^2 + \vec{a}^2 R^4 - 2R^2 (\vec{R} \cdot \vec{a})^2) \\ &= \frac{e^2}{c^4 R^4} (R^2 \vec{a}^2 - (\vec{R} \cdot \vec{a})^2) = \frac{e^2 \vec{a}^2}{c^4 R^2} (1 - \cos^2 \theta) \end{aligned}$$

$$\vec{S}_a = \frac{c}{4\pi} \frac{e^2 \vec{a}^2 \sin^2 \theta}{c^4 R^2} \frac{\vec{R}}{R} = \frac{e^2 \vec{a}^2 \sin^2 \theta}{4\pi c^3 R^2} \vec{R}$$

Express in terms of the angular distribution.

$$\text{total power} = P = \oint \vec{S}_a \cdot d\vec{A} = \oint \underbrace{\vec{S}_a \cdot \hat{R}}_{= dP/d\Omega} R^2 \sin\theta d\theta d\phi$$

$$\frac{dP}{d\Omega} = \frac{e^2 a^2}{4\pi c^3} \sin^2 \theta$$

note: this is evaluated at the particle's local time

Θ is measured from \vec{a}

$dP/d\Omega \rightarrow 0$ at $\Theta=0$ (forward)

\rightarrow max at $\Theta=\pi/2$ transverse.

total rad. power:

$$P = \int \left(\frac{dP}{d\Omega} \right) d\Omega = 2\pi \int_0^\pi \frac{e^2 a^2}{4\pi c^3} \sin^2 \theta \sin \theta d\theta$$

$$= \frac{e^2 a^2}{2c^3} \int_0^\pi \sin^3 \theta d\theta$$

$$\boxed{P = \frac{2}{3} \frac{e^2 a^2}{c^3}}$$

Larmor radiation formula.
remember this!

Qualitative analysis of radiation from impulsive acceleration (ct)

$$\frac{1}{2} a \tau^2$$

initial velocity not important

observe at large distance

kink in \vec{E} propagates out at c

$$\text{final velocity } u = a\tau$$

$$\text{compare } \frac{1}{2} a \tau^2 \text{ to } c\tau \rightarrow \frac{1}{2} a \tau < c$$

trace kink back to source:



$$\frac{E_\perp}{E_\parallel} \sim \frac{\frac{1}{2} a \tau^2}{c\tau} = \frac{a\tau}{2c}$$

$$E_\parallel \text{ at start of accel} \sim \frac{q}{r^2} \text{ w/ } r \sim c\tau$$

$$E_\perp \sim \frac{a\tau}{2c} \cdot \frac{q}{r^2} \text{ elim. } \tau \rightarrow \frac{qa}{2c^2} \cdot \frac{1}{r}$$

If \vec{a} is not \perp to \vec{r}



$\rightarrow \sin \theta$ factor

Radiation w/ velocity // acceleration

$$\vec{E}_a = \frac{e}{c^2 K^3 R^3} \vec{R} ((\vec{R} - \vec{\beta} R) \times \vec{a})$$

↓
o since here $\vec{\beta} \times \vec{a} = 0$

now $\vec{S}_a = \vec{S}_a (\beta=0) \cdot K^{-6}$

but this is power radiated from charge:

$$\frac{dE_{tot}}{dt} = - \int \vec{S}_a \cdot d\vec{A}$$

we want radiated power measured at distant point,

$$P = - \frac{dE_{tot}}{dt'} = - \frac{dE}{dt} \frac{dt}{dt'} \quad t' = t - R(t)/c$$

→ retarded time

$$\frac{dt}{dt'} = 1 + \frac{1}{c} \frac{d}{dt'} |\vec{r} - \vec{r}_e(t)| = 1 + \sum \frac{q_i}{c} \frac{d}{dx_{ei}} |\vec{r} - \vec{r}_e|$$

$$|\vec{r} - \vec{r}_e| = \sqrt{r^2 + r_e^2 - 2\vec{r} \cdot \vec{r}_e}$$

$$\frac{d}{dx_{ei}} |\vec{r} - \vec{r}_e| = \frac{1}{2} \frac{1}{\sqrt{r^2 + r_e^2 - 2\vec{r} \cdot \vec{r}_e}} (2x_{ei} - 2x_e)$$

$$\sum \beta_i \frac{d}{dx_{ei}} |\vec{r} - \vec{r}_e| = \sum \beta_i \frac{(x_{ei} - x_e)}{R} = - \frac{\vec{\beta} \cdot \vec{R}}{R}$$

$$\frac{dt}{dt'} = 1 - \frac{\vec{\beta} \cdot \vec{R}}{R} = K = 1 - \beta \cos \theta$$

→ measured rel to \vec{u}

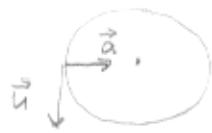
now we have K^{-5} in $dP/d\Omega$

this directs lobes toward \vec{u}



Synchronization condition - circular orbit.

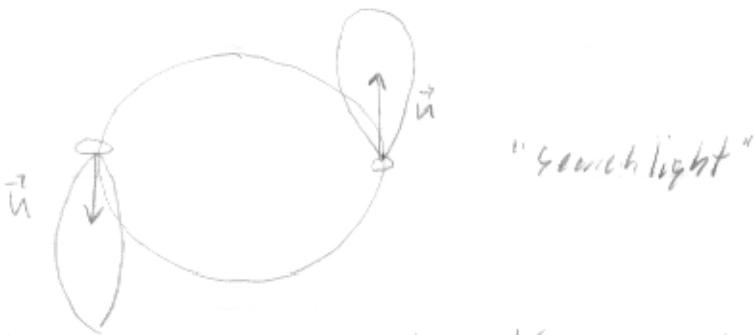
$$\vec{u} + \vec{a}$$



At low speeds, Larmor frequency w/ θ relative to \vec{a}



higher speeds: push lobes forward, toward \vec{u}



output pulses w/ repeat rate $1/\text{orbit period}$