Nonlinear Optics
Homework 4
due Wednesday, 11 Feb 2009

revised with clarifications 9 Feb 2009

- **Problem 1:**
  The impulse response for a classical damped harmonic oscillator is
  h[t] = Exp[- γ t] Sin[ω0s t] for t > 0, and h[t] = 0 for t<0
  where ω0s is a slightly shifted resonance frequency defined by
  $\omega 0s = \sqrt{\omega 0^2 - \gamma^2}$

  Use transform theorems to calculate the corresponding transfer function:
  H(ω) = FT{h(t)}
  to show that you recover what is expected from the linear solution described in Chapter 1. You may check your result by doing the transform in *Mathematica*, but I want the work to be done analytically. You may find the pdf of Fourier Transform theorems useful (see website).

- **Problem 2: Doubling of chirped pulses**
  a. By representing a linearly-chirped Gaussian pulse in the time domain (see notes), calculate the ideal frequency doubling signal, that is, where there is no phase mismatch or depletion. In this case
  $E_2(t) \propto E_1{}^2(t)$

  b. Next transform $E_2(t)$ to the spectral domain. Instead of doing this from scratch, scale the expression for the transform of the chirped pulse $E_1(\omega)$ to $E_1(t)$. From the expression for $E_2(\omega)$, determine the spectral width of the doubled pulse, and from this calculate its transform-limited pulse duration. The transform-limited pulse duration is the duration you obtain when there is no spectral phase (technically, when there is no curvature to the spectral phase).

  c. Compare your results for the output spectral and transform-limited widths to what you get if you just assume that the temporal profile of the input pulse looks just like the input spectrum. In the latter case, you obtain the output spectrum directly by squaring the input spectrum.

- **Problem 3: opposite chirp sum frequency mixing of pulses**
  This problem is similar to the previous problem except that instead of directly doubling a single input beam, we do sum frequency mixing of two pulses that are combined with equal but opposite-sign linear chirp.
  a. Show that the output pulse $E_2(t)$ is transform-limited, with a duration equal to $1 \big/ \sqrt{2}$ times the duration of the chirped input pulse. This is one way to efficiently make narrowband pulses from ultrashort pulses.
  b. Now introduce a relative time delay T between the pulses (the math works out easier if you shift them in opposite directions by T/2). Describe the nature of the output pulse in this case as a function of the delay T: temporal shape and width, peak amplitude, instantaneous frequency characteristics.

- **Problem 4: OPA of short pulses**
  In an OPA the gain in the undepleted limit is exponentially dependent on the pump pulse intensity (see discussion in section 2.8). In practice, phase matching limits the output pulse duration, but for this problem, assume that Δk=0, and make an estimate the output pulse duration using techniques similar to the previous problems. That is, work in the non-depleted limit (signal and idler grow exponentially rather than sinh( ) and cosh( )). Assume if the peak parametric gain is 10^6 and that the pump pulse is unchirped and has a Gaussian temporal profile. Hint: you can expand the pump pulse around its peak to get a simpler form.