Harmonic generation and optical parametric amplifiers.

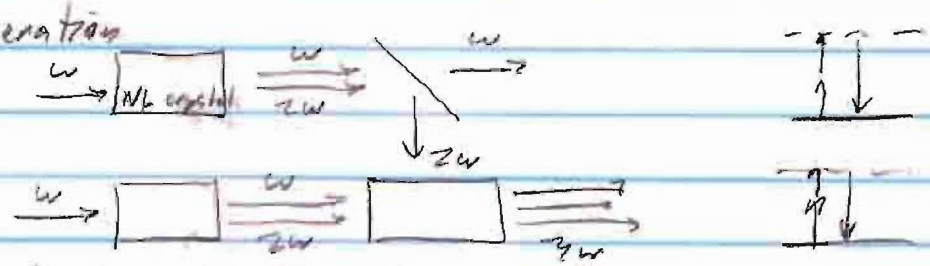
Motivation:

- Many applications (industrial, commercial, scientific) require specific wavelengths
- cost-effective laser sources are limited
  - Nd:YAG, ti:sapphire, other solid state.
  - laser diodes
  - gas lasers: HeNe, ion (Ar^+, Kr^+), excimer
  - dye lasers
- best to get one laser, convert λ

Harmonic generation

SHG

THG



direct THG difficult to phase match.

4HG

Nd:YAG λ's

$\lambda_1 = 1064\,nm$

$\lambda_2 = 532\,nm$

$\lambda_3 = 355\,nm$

$\lambda_4 = 266\,nm$

Issues:

how to get good efficiency?

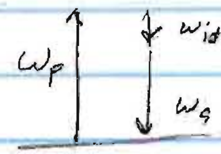
- intensity, crystal length, focusing
- phase matching: birefringent / angle tune, quasi-phase matching

Parametric mixing

└ no absorption, instantaneous response.

sum mixing $\omega_3 = \omega_1 + \omega_2$

OPA $\omega_{signal} = \omega_{pump} - \omega_{idler}$



requires a seed, phase matching

difference-frequency mixing

- same process, ~ equivalent energy in ω_p, ω_s

can tune across wide λ range, especially with mixing of OPA output.

		OPA	SHG
example: pump 800 nm			
	sig	1 μm → 1.4 μm	500 nm → 700 nm
	idler	3 μm → 1.6 μm	1500 nm → 800 nm

Nonlinear wave propagation.

- go from microscopic to macroscopic.

As before, we use Maxwell eqns to get wave eqn

$$\underbrace{\nabla\times\nabla\times\vec{E}}_{\nabla(\nabla\cdot\vec{E})-\nabla^2\vec{E}} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = -\frac{4\pi}{c}\frac{\partial^2\vec{P}}{\partial t^2} \quad \overset{SI}{\boxed{= -\frac{1}{\varepsilon_0 c^2}\frac{\partial^2\vec{P}}{\partial t^2}}}$$

$$\nabla\cdot\vec{D} = \nabla\cdot(\varepsilon\vec{E}) = 0$$

We'll assume ε ~ spatially constant, even though there is nonlinearity

Next separate $\vec{P} = \vec{P}^{(1)} + \vec{P}^{NL}$, bring linear part over.

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\underbrace{(\vec{E} + \overset{\frac{1}{\varepsilon_0}}{4\pi}\vec{P}^{(1)})}_{=\vec{D}^{(1)} = \varepsilon^{(1)}\vec{E}} = \frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\vec{P}^{NL} \quad \boxed{= \frac{1}{\varepsilon_0 c^2}\partial_t^2\vec{P}^{NL}}$$

→ assume isotropic

assume this is t-indep.

$$\nabla^2\vec{E} - \frac{\varepsilon^{(1)}}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2\vec{P}^{NL}}{\partial t^2} \quad \boxed{= \frac{1}{\varepsilon_0 c^2}\partial_t^2\vec{P}^{NL}}$$

solutions:

linear eqn has wave solutions.

we will make the assumption that there will be several diff4 waves at distinct frequencies.

$$\text{e.g.}\quad \vec{E}_n(\vec{r},t) = E_n(\vec{r})e^{-i\omega_n t} + c.c.$$

$$\vec{E}(\vec{r},t) = \sum_n \vec{E}_n(\vec{r},t) \qquad n \geq 0$$

similar for $\vec{P}_n^{NL}$, and $\vec{D}_n^{(1)} = \varepsilon_n^{(1)}\vec{E}_n^{(1)}$

$$\rightarrow -\nabla^2 \vec{E}_n(r) - \frac{\omega_n^2}{c^2}\varepsilon^{(1)}(\omega_n)\vec{E}_n(\vec{r}) = \frac{4\pi\omega_n^2}{c^2}\vec{P}_n^{NL}(\vec{r}) = \frac{\omega_n^2}{\varepsilon_0 c^2}\vec{P}_n^{NL}(\vec{r})$$

What if material is birefringent?

$$\vec{D}_n^{(1)} = \overleftrightarrow{\varepsilon}^{(1)}(\omega_n)\cdot\vec{E}_n^{(1)} \quad \text{since } \varepsilon \text{ is a tensor}$$

$\therefore$ change $\varepsilon \rightarrow \overleftrightarrow{\varepsilon}$ and dot it with $\vec{E}$

Notes: we have several coupled equations, each at diff't ω_n

- coupling is through $\vec{P}^{NL}$

e.g. $P^{(2)} = \chi^{(2)} E_m E_n^*$

each of these equations is for 3 vector components.

Application: SFG (sum freq. gen.)

- cw
- 2 inputs at ω_1, ω_2
- outputs at $\omega_3 = \omega_1 + \omega_2$
- plane waves prop in z direction

- ignore polarization effects for now

$$E_n(z,t) = A_n e^{i(k_n z - \omega_n t)} + c.c.$$

with no nonlinearity, no coupling of separate, linear w.e.

$\therefore$ all A_n's are constant

with nonlinearity, RHS $\rightarrow$ P^{NL} terms, e.g.

$$P_3^{(2)} = 4 d_{eff} E_1 E_2 \qquad \text{recall} \quad \chi^{(2)} \equiv 2 d_{eff}$$

$$= 4 d_{eff} A_1 A_2 e^{i(k_1+k_2)z}$$

Second 2x from $\omega_1 + \omega_2$ or $\omega_2 + \omega_1$

to put this into the NLWE, we have to account for $A_n(z)$, no dependence on x, y

$$-\frac{d^2}{dz^2}\vec{E}_3(z) - \frac{\omega_3^2}{c^2}\varepsilon_3 \vec{E}_3(z) = \frac{16\pi\omega_3^2}{c^2} d_{eff}\vec{E}_1(z)\vec{E}_2(z)$$

$$\boxed{= \frac{4\omega_3^2}{c^2} d_{eff}\vec{E}_1(z)\vec{E}_2(z)}$$

Non-depleted pump approximation:

- input beams have powers $\propto A_1^2, A_2^2$
- anticipate growth of A_3 from zero, no initial change (much) of pump beams. A_1, A_2 const.

$\therefore$ equations are decoupled.

$$\rightarrow -\left(\frac{d^2}{dz^2}A_3\right)e^{ik_3z} - 2ik_3\left(\frac{d}{dz}A_3\right)e^{ik_3z} + \cancel{k_3^2 A_3 e^{ik_3z}} - \cancel{\frac{\varepsilon_3\omega_3^2}{c^2}A_3 e^{ik_3z}}$$

$$= \frac{16\pi\omega_3^2}{c^2} d_{eff} A_1 A_2 e^{i(k_1+k_2)z}$$

cancellation from $k_3^2 = \varepsilon_3\omega_3^2/c^2$

notice where e^{ik_3z} is: only on LHS

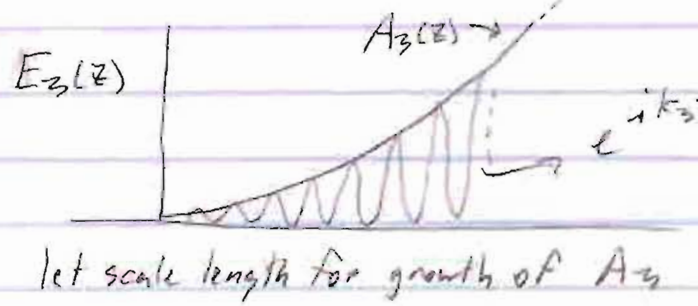
divide it out:

$\boxed{\frac{4\omega_3^2 d_{eff}}{c^2}}$

$$-\frac{d^2A_3}{dz^2} - 2ik_3\frac{dA_3}{dz} = \frac{16\pi\omega_3^2}{c^2} d_{eff} A_1 A_2 e^{i\Delta kz}$$

where $\Delta k \equiv k_1 + k_2 - k_3$ = phase mismatch.

Can we drop d^2A_3/dz^2 term?



let scale length for growth of A_3 be l

$\rightarrow \frac{dA_3}{dz}$ scales as $\frac{1}{l}A_3$ e.g. $A_3 = A_{30}e^{z/l}$

compare terms:

$\frac{1}{l^2}A_3 : \frac{2\pi}{\lambda l}A_3$ if $\lambda l \ll l^2$ or $\lambda \ll l$

we can drop d^2A_3/dz^2 $\rightarrow$ slowly varying envelope approx.

slowly varying amplitude approx (SVEA)

$$\left|\frac{d^2A_3}{dz^2}\right| << \left|k_3 \frac{dA_3}{dz}\right|$$

$$\rightarrow \frac{dA_3}{dz} = \frac{8\pi i\, d_{eff}\, \omega_3^2}{k_3 c^2} A_1 A_2 e^{i\Delta k z}$$

$\boxed{2 i\, d_{eff}}$

$$= \frac{2\pi i \omega_3}{n_3 c} p_3 e^{i\Delta k z} \qquad p_3 \equiv 4 d_{eff} A_1 A_2$$

$\frac{2\pi i \omega_3}{n_3 c} \rightarrow \boxed{\frac{i\omega_3}{n_3 c}}$

Account for pump depletion:

- let $A_1(z)$, $A_2(z)$
- eqn for dA_3/dz is unchanged.
- now do same for others - main work is getting RHS = driving terms

$$P_1^{(2)} = 4 d_{eff} E_2^* E_3 \qquad \omega_1 = \omega_3 - \omega_2$$

$\uparrow$ conjugate E_2

$$\rightarrow \frac{dA_1}{dz} = \frac{8\pi i\, d_{eff}\, \omega_1^2}{k_1 c^2} A_2^* A_3 \underbrace{e^{i(k_3-k_2-k_1)}}_{e^{-i\Delta k z}}$$

$\boxed{2 i d_{eff}}$

$$\frac{dA_2}{dz} = \frac{8\pi i\, d_{eff}\, \omega_2^2}{k_2 c^2} A_1^* A_3 e^{-i\Delta k z}$$

$\boxed{2 i d_{eff}}$

here, 3 eqns 3 unknowns (actually 6 eqns, b unk. since A_i's are complex)

We'll come back to solution later.

Deal with simpler cases first:

1) non depletion (1 eqn) $\Delta k = 0$
2) non depl. $\Delta k \neq 0$
3) depletion (all 3) $\Delta k = 0$
4) " $\Delta k \neq 0$
5) add dispersion...