

## Fourier Transforms: Transform pairs, theorems in t, ω domain

Definitions and theorems (in Mathematica, use FourierParameters->{1,1}):

**Forward transform:**  $\mathfrak{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$

**Inverse transform:**  $\mathfrak{F}^{-1}\{F(\omega)\} = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega$

**Shift Theorem:**  $\mathfrak{F}\{f(t - t_0)\} = \exp(+i\omega t_0) F(\omega)$        $\mathfrak{F}^{-1}\{F(\omega - \omega_0)\} = \exp(-i\omega_0 t) f(t)$

**Scale Theorem:**  $\mathfrak{F}\{f(at)\} = \frac{1}{|a|} F(\omega/a)$        $\mathfrak{F}^{-1}\{F(b\omega)\} = \frac{1}{|b|} f(t/b)$

**Conjugate:**  $\mathfrak{F}\{f^*(t)\} = F^*(-\omega)$

**Inverse transform pair:**  $\mathfrak{F}\{F(t)\} = 2\pi f(-\omega)$        $\mathfrak{F}^{-1}\{f(\omega)\} = \frac{1}{2\pi} F(-t)$

**Convolution:**  $h(t) = f(t) \otimes g(t) = \int_{-\infty}^{\infty} f(t') g(t - t') dt'$

**Convolution w/delta fcn:**  $\delta(t - t_0) \otimes f(t) = f(t - t_0)$

**Convolution theorem:**

$$f(t) \otimes g(t) = \mathfrak{F}^{-1}\{F(\omega)G(\omega)\} \quad \mathfrak{F}\{f(t)g(t)\} = \frac{1}{2\pi} F(\omega) \otimes G(\omega)$$

**Parseval's theorem (conservation of energy):**  $\int |f(t)|^2 dt = \frac{1}{2\pi} \int |F(\omega)|^2 d\omega$

### Transform pairs:

**Delta functions:**

$$\mathfrak{F}\{e^{\pm i\omega_0 t}\} = \int_{-\infty}^{\infty} e^{\pm i\omega_0 t} e^{i\omega t} dt = 2\pi \delta(\omega \pm \omega_0) \quad \mathfrak{F}^{-1}\{e^{\pm i\omega t_0}\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\pm i\omega_0 t} e^{-i\omega t} dt = \delta(t \mp t_0)$$

**Gaussian:**

$$\mathfrak{F}\{\exp(-t^2/t_w^2)\} = \sqrt{\pi} t_w^2 \exp(-t_w^2 \omega^2/4)$$

**Rect function** (rect(u) = 1 for  $-\frac{1}{2} < u < \frac{1}{2}$ ):

$$\mathfrak{F}\{\text{rect}(t/t_0)\} = t_0 \text{sinc}(\omega t_0/2)$$

$$\mathfrak{F}\{\text{sinc}(t/2t_0)\} = 2\pi t_0 \text{rect}(\omega t_0)$$

**Cosine function:**

$$\mathfrak{F}\{\cos(\omega_0 t)\} = \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

**Array comb**( $t/t_0$ )  $\equiv \sum_{n=-\infty}^{\infty} \delta(t - nt_0)$ :

$$\mathfrak{F}\{\text{comb}(t/t_0)\} = 2\pi \text{comb}[\omega/(2\pi/t_0)]$$