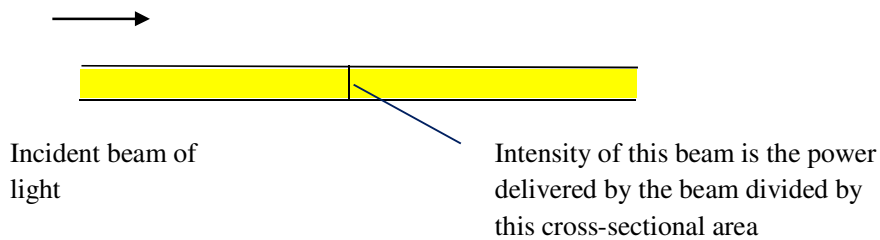
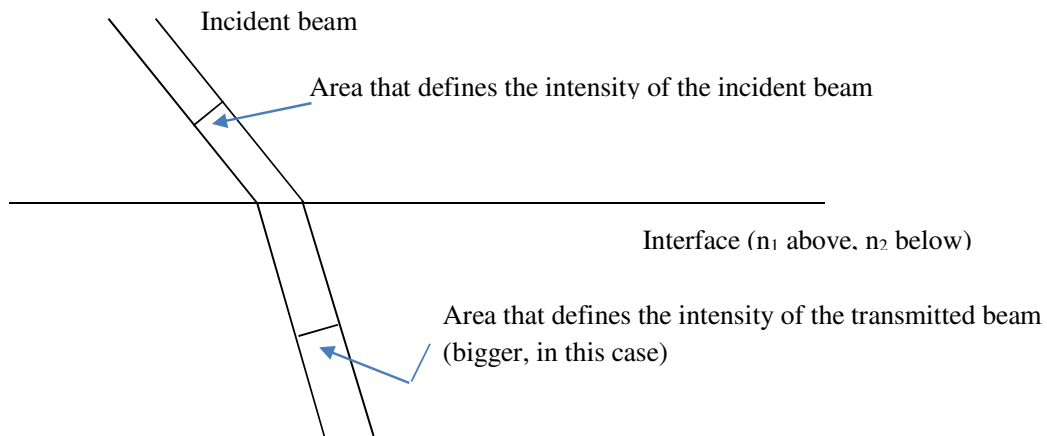


PHGN 462 Homework 7

1) Non-normal incidence has one other tiny little complication we haven't talked about yet. So far we've been using the definitions  $R = \frac{I_R}{I_I}$  and  $T = \frac{I_T}{I_I}$ , and power conservation has required that  $\frac{I_R}{I_I} + \frac{I_T}{I_I} = 1$ . But if you use the Fresnel equations to generate those intensities in the general, non-normal case and calculate  $R$  and  $T$ , they don't add up to 1. That's bad. Power should still be conserved in non-normal incidence problems. The issue is that intensity as we've defined it is power per unit area *assuming normal incidence*:



When light is incident on a surface at some angle, we still define intensity in the same way, but the width of a particular slice of wave *changes* as we go from incident to transmitted:



- a) In order to preserve the core principle that power in equals power out for some particular ray, we need to use geometry to generalize the equation  $\frac{I_R}{I_I} + \frac{I_T}{I_I} = 1$ . Figure out how. You're going to need to add some trig factors here and there.

- b) Using the above and the Fresnel equations, show  $r + t = 1$ , where  $r$  and  $t$  are the new versions of  $R$  and  $T$  that properly respect the geometry of the non-normal case. To keep it simple, just do the TE case, and let  $n_1 = 1$ .

2) We know enough to start to explain a lot of neat optical phenomena that we see in the world around us. A few weeks ago, my wife and I went for a walk by the creek. We were wearing sunglasses, and noticed that the reflection of light off the water changed dramatically if we tilted our heads. We were able to faithfully capture the effect on video, as seen in the following:

<https://youtu.be/5lMw-xTh8PI>

<https://youtu.be/SG2xUYY-mEw>

The two videos involve two different pairs of sunglasses, and there's some extra information in the video descriptions.

Construct an explanation for why we saw what we saw. There are a lot of factors that *might* contribute; make sure your explanation accounts for as much of what we observe as possible, in detail. It's possible you'll need to look up supplementary information. The complete answer might not be obvious. I mean that literally – I'm not totally sure I have it myself, and I'm looking forward to seeing what people come up with. You're welcome to ask me questions about the circumstances.

Note in particular that I've checked each pair of sunglasses against a polarizing filter and discovered that neither is linearly polarized.

3) (From Pollack and Stump 13.9)

a) From first principles, set up the boundary conditions for a plane wave incident normally on the surface of a conductor. Let the incident wave travel in the  $x$  direction, and be polarized in the  $y$  direction. The reflected and transmitted waves travel in the  $-x$  and  $+x$  directions, respectively. The wave vector  $\kappa$  in the conductor is complex, and the dispersion relation is:

$$\kappa^2 = +i\mu\sigma\omega + \mu\epsilon\omega^2$$

Solve the boundary conditions, and show that the amplitude of the reflected wave is correctly given by:

$$\frac{E_0''}{E_0} = \frac{1 - n_2}{1 + n_2} = \frac{\omega - c\kappa}{\omega + c\kappa}$$

b) For a good conductor, i.e.,  $\sigma \gg \epsilon\omega$ , derive:

$$R \approx 1 - \frac{4\omega c\kappa_1}{c^2(\kappa_1^2 + \kappa_2^2)} = 1 - \sqrt{\frac{8\omega\epsilon_0}{\sigma}}$$

c) Show that for a good conductor, at the surface the field of the reflected wave is approximately equal but opposite to the field of the incident wave. You may take the dispersion relation from part a as given. Also, you may assume the conductor is not significantly magnetic.

Note that this problem is largely done in Pollack and Stump for you if you'd like to check it out (chapter 13.3). Then it's just a matter of going through the steps yourself and filling in the gaps.