MATH348 - July 2, 2009 **NAME**:

Exam I - 50 Points

In order to receive full credit, SHOW ALL YOUR WORK. Full credit will be given only if all reasoning and work is provided. When applicable, please enclose your final answers in boxes.

1. (10 Points) True/False and Short Response

   (a) Mark each statement True or False. When not explicitly stated take $\mathbf{A} \in \mathbb{R}^{m \times n}$.

      i. If a matrix $\mathbf{A}$ has a row of zeros then $\mathbf{Ax} = \mathbf{0}$ has infinity-many solutions.

      ii. If $\mathbf{A}$ has a pivot in every row then $\mathbf{Ax} = \mathbf{b}$ has a solution.

      iii. If $\mathbf{A}$ has a pivot in every column then $\mathbf{Ax} = \mathbf{b}$ has a solution.

      iv. The system $\mathbf{Ax} = \mathbf{0}$, where $\mathbf{A} \in \mathbb{R}^{3 \times 5}$, has only the trivial solution.

   (b) Please respond to the following questions and justify your position:

      i. Suppose that $\det(\mathbf{A}) = 0$, what can be said about the dimension of the null-space of $\mathbf{A}$ and the dimension of the column-space of $\mathbf{A}$?

      ii. Is it possible for a vector to be in both the null-space and the column-space of a matrix?

2. (10 Points) Quickies:

(a) Given,

$$\mathbf{A}_1 = \begin{bmatrix} 8 & 0 & 8 \\ 0 & 8 & 0 \\ 8 & 0 & 8 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 8 & 0 \\ 8 & 0 & 8 \\ 0 & 8 & 0 \end{bmatrix} \quad \mathbf{b}_1 = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix} \quad \mathbf{b}_3 = \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$$

Do solutions to the following equations exist and are these solutions unique? (Yes or No)

Equation	Solutions Exist	Solutions are Unique
$\mathbf{A}_1\mathbf{x} = \mathbf{b}_1$		
$\mathbf{A}_2\mathbf{x} = \mathbf{b}_2$		
$\mathbf{A}_1\mathbf{x} = \mathbf{b}_1 + \mathbf{b}_3$		
$\mathbf{A}_2\mathbf{x} = \mathbf{b}_1 - \mathbf{b}_3$		
$(\mathbf{A}_1 + \mathbf{A}_2)\,\mathbf{x} = \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$		

(b) Let $\mathbf{A} = \begin{bmatrix} 1 & h \\ 3 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ h \end{bmatrix}$. Find **all** values h for which the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ is:

i. Consistent with a unique solution.

ii. Consistent with infinity-many solutions.

iii. Inconsistent.

(c) Given,

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}. \tag{1}$$

Find one eigenvalue of $\mathbf{A}$.

3. (10 Points) Given,

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}.$$

For what values of h are the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a linearly dependent?

4. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}.$$

Find all matrices associated with the diagonal decomposition, $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$, of $\mathbf{A}$.

5. (10 Points) Given,

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & \mathbf{1} & 0 \\ 0 & 0 & \mathbf{1} & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 \end{bmatrix} \tag{2}$$

Find a basis and the dimension of the null-space and column-space of $\mathbf{A}$.

6. (Extra Credit 1) Given,

$$[\mathbf{A}|\mathbf{0}] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]. \tag{3}$$

List as many properties/characterizations of this system as you can for one point each and six points maximum.

7. (Extra Credit 2) Concerning the matrix in problem 5 of this exam.

   (a) Is the matrix invertible? Justify your choice.

   (b) Can only one element of the matrix be changed so that its invertibility is altered?

   (c) Is this matrix happy?

   (d) Can you change the mood of this matrix with one elementary row-step?