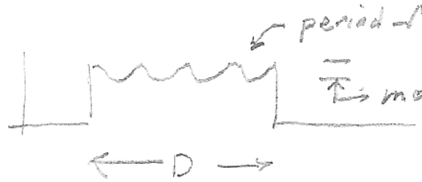
Fraunhofer diffraction: examples + applications.

sinusoidal transmission gratings

- some diff'n gratings are produced by recording interference fringes:

photoresist: expose, etch, coat.

types: absorbing $T(x) =$



$$= rect(x/D)\left(1 - A + A\cos\left(\tfrac{2\pi x}{\Lambda}\right)\right)$$

phase $\quad T(x) = rect(x/D)\exp\left(i k_0 n(x) l\right)$

$$w/\ n(x) = n_0 + n_m \cos\left(\tfrac{2\pi x}{\Lambda}\right)$$

with $k_0 n_m l \ll 1$

$$T(x) \doteq rect(x/D)\, e^{i k_0 n_0 l}\left(1 + i k_0 n_m l \cos(\,)\right)$$

imag!

reflection = phase

$$R(x) = rect(x/D)\, e^{i k_0 l_m \cos\left(\frac{2\pi}{\Lambda}x\right)}$$

In any case, transmission is of the form

$$T(x) = a + b\cos\left(\tfrac{2\pi}{\Lambda}x\right)$$

note that $2\pi/\Lambda$ is a type of wavevector. Let $k_{xm} = 2\pi/\Lambda$

if incident wave is $e^{ik_0 z}$ wave just past the grating is

$$a e^{ik_0 z} + \tfrac{1}{2} b\left(e^{i(k_{xm}x + k_0 z)} + e^{i(-k_{xm}x + k_0 z)}\right)$$

sum of 3 waves:

$\sim e^{i(k_{xm}x + k_0 z)}$ (+1 order)

$\sim e^{ik_0 z}$ (zero order)

$\sim e^{i(-k_{xm}x + k_0 z)}$ (−1 order)

k_{xm} is a spatial frequency $\doteq k_0 \sin\theta_m$
plane wave traveling at angle θ_m

side note: outgoing wave really should be of the form

$$\exp(i k_0 (x \sin\theta_m + z \cos\theta_m))$$

to conserve $\vec{k}$ (which is like photon momentum, $\hbar\vec{k}$)
But we are implicitly looking at small angles: $\cos\theta_m \sim 1$.

Do diffr. integral:

$$E_{diff}(X,Y) = \frac{e^{ik_0R}}{i\lambda R} \iint_{-\infty}^{\infty} E_{in}(x,y)\, T(x,y)\, e^{-i\left(\frac{k_0 X}{R}x + \frac{k_0 Y}{R}y\right)} dx\,dy$$

or

$$= \frac{e^{ik_0R}}{i\lambda R} \mathcal{F}_{xy}\{E_{in}(x,y)\,T(x,y)\} \quad \text{with } \beta_x = \frac{k_0 X}{R},\ \beta_y = \frac{k_0 Y}{R}$$

Here $E_{in}(x,y) = E_0$ at $z=0$

$$T(x,y) = \text{rect}(x/D)\,(a + b\cos(k_{mx}x))\,\text{rect}(y/D)$$

for a square grating

$$= T_x(x)\,T_y(y)$$

$$E_{diff}(X,Y) = \frac{e^{ikR}}{i\lambda R} \mathcal{F}_x\{T_x(x)\}\, \mathcal{F}_y\{T_y(y)\}$$

notice $T(x,y)$ is separable, no convolution because x, y are different spaces.

$$E_{diff}(\ ) = \frac{e^{ikR}}{i\lambda R} \left(\frac{D}{2\pi}\text{sinc}\left(\frac{\beta_x D}{2}\right) \otimes \left(a\, 2\pi\,\delta(\beta_x) + \frac{b}{2} 2\pi\left(\delta(\beta_x + k_{mx}) + \delta(\beta_x - k_{mx})\right)\right)\right.$$

$$\left. \cdot \left(D\,\text{sinc}\left(\frac{\beta_y D}{2}\right)\right)\right)$$

recall that β_x variable is like $k_0 \sin\theta_x = k_0 \frac{X}{R}$
so physical meaning of $\delta(\beta_x + k_{mx})$ is that the wave has a well-defined angle:

$$k_0 \sin\theta_x + k_{mx} = 0 \quad \text{at} \quad \sin\theta_x = -\frac{k_{mx}}{k_0}$$

$$\text{or} \quad \sin\theta_x = -\frac{2\pi}{-\Lambda}\frac{\lambda}{2\pi} = (-1)\lambda/\Lambda \quad \text{like grating eqn.}$$

carry out convolution:

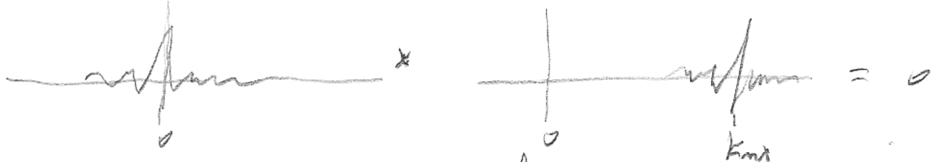
$$E_{diff}(X,Y) = \frac{e^{ikR}}{i\lambda R} D^2 \left[a\,\mathrm{sinc}\left(\frac{\beta_x D}{2}\right) + \frac{b}{2}\left(\mathrm{sinc}\left(\frac{(\beta_x + k_{mx})D}{2}\right) + \mathrm{sinc}\left(\frac{(\beta_x - k_{mx})D}{2}\right)\right)\right] \cdot \mathrm{sinc}\left(\frac{\beta_y D}{2}\right)$$

We can separate out the physical effects of the grating, which splits beam into diff't orders (angles) and the aperture of the grating, which → sinc( ) envelope.

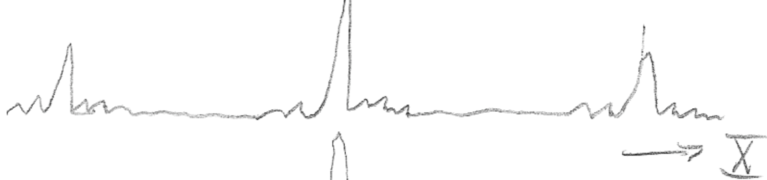
Add lens: $\beta_x = \frac{k_0 X}{R} \rightarrow \frac{k_0 X}{f}$

in focal plane, measure intensity: $|E_{diff}|^2$
- if peaks are separate, cross terms aren't impt.

e.g. $\mathrm{sinc}\left(\frac{\beta_x D}{2}\right)\mathrm{sinc}\left(\frac{(\beta_x + k_{mx})D}{2}\right) \approx 0$



Focal plane is $Y = 0$:



$X = 0$: