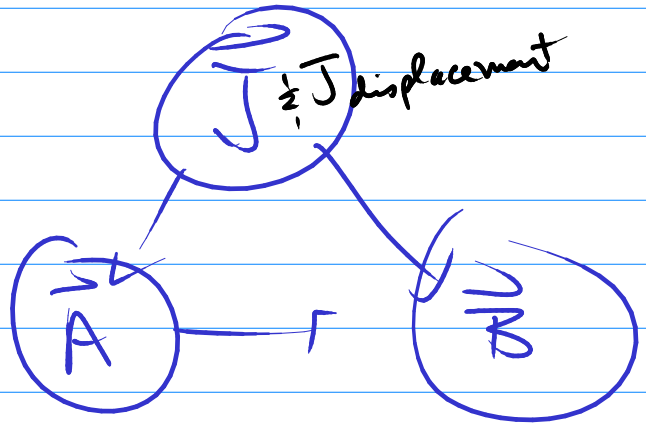


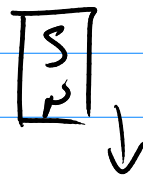
generates a
 $J_{\text{displacement}} \Rightarrow$ magnetic field

Does $J_{\text{displacement}}$ go into
 our expression for conservation
 of charge? **NO**

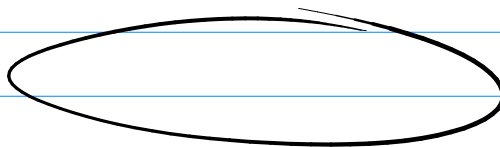
$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$



Homework problem on last
 set



See soln on wiki

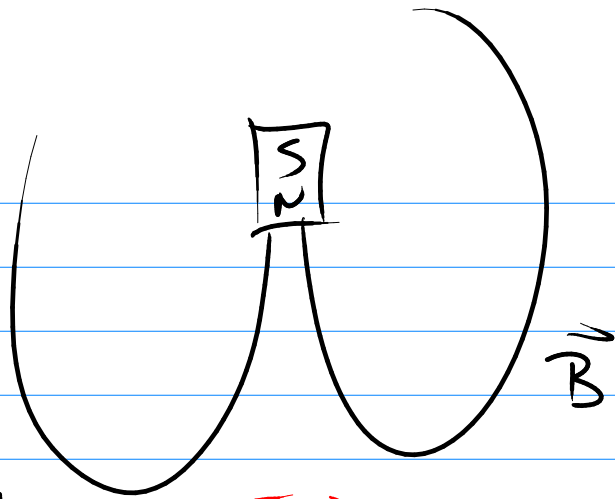


$$\mathcal{E}_{\text{emf}} = -\frac{d\Phi_B^{\text{tot}}}{dt}$$

$$\Phi_B^{\text{tot}} = \int \vec{B} \cdot d\vec{a}$$

$$\vec{B}_{\text{wire}} + \vec{B}_{\text{magnet}}$$

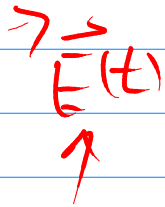
$$\Phi_{\text{wire}} = \int B_{\text{wire}} da \equiv LI$$



Should also include B due to

J displacement but it is small (see below)

no metal



causes $\vec{J}_{\text{displ}} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{B}$

Maxwell's Eqns

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

2 sources of B
 $\vec{J} \pm \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

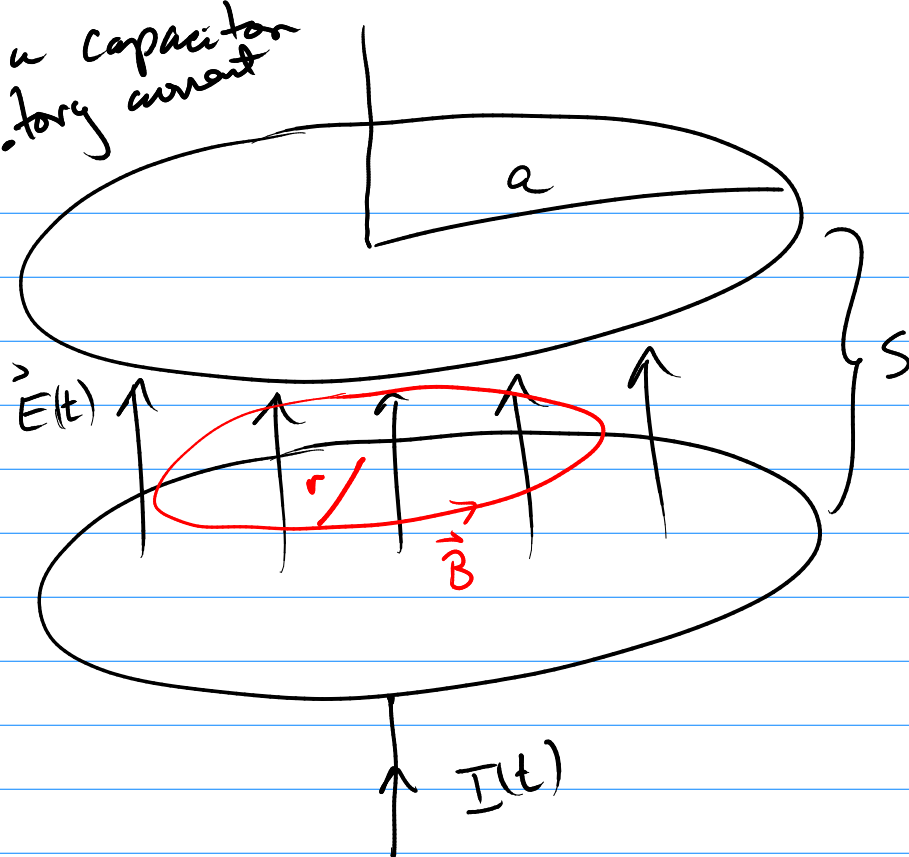
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

2 sources of E
 charge $\pm \frac{\partial \vec{B}}{\partial t}$

conservation \pm non-E

charging a capacitor
with oscillatory current



$$\underline{\underline{A = \pi a^2}}$$

$$\vec{J}_{\text{displacement}} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$E = \frac{\sigma}{\epsilon_0} = \frac{1}{\epsilon_0} \frac{Q}{A}$$

$$\vec{J}_{\text{displacement}} = \frac{I}{A} \hat{z}$$

$$\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0} \frac{I}{A}$$

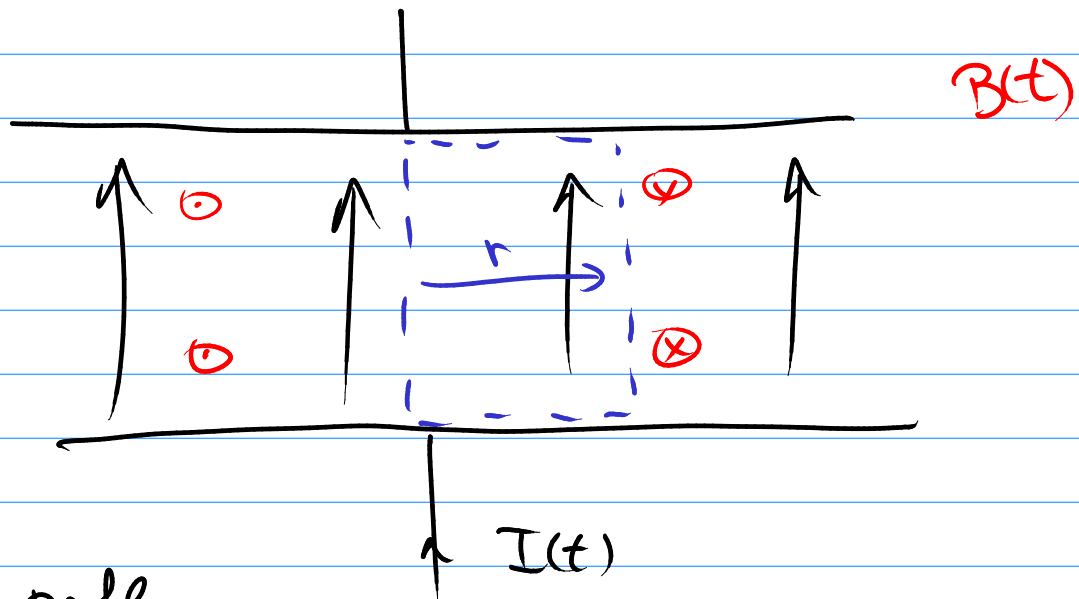
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int \vec{J}_d \cdot d\vec{a} = \mu_0 \frac{I}{\pi a^2} \pi r^2$$

$$B \cdot 2\pi r = \mu_0 I \frac{r^2}{a^2} \Rightarrow \vec{B} = \frac{\mu_0 I r}{2\pi a^2} \hat{\phi}$$

If $Q = I_0 t$ then $I = I_0$ is constant
and B is constant. No need to include any
effects of a changing B generating another E (Faraday's law).

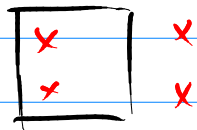
If $I(t)$ then $B(t)$.

side view



Analogous problem

$I(t)$



$$\mathcal{E}_{\text{ind}} = - \frac{d\Phi_B}{dt}$$

$$\oint \vec{E} \cdot d\vec{e} = - \int \frac{d(\vec{B})}{dt} \cdot d\vec{a}$$

Assume harmonic oscillation

$$E_1 e^{i\omega t}$$

$$E = E_1 + E_2 + \dots$$

1st term in perturbation not uniform

uniform (constant)

$$\frac{\sigma_0}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{e} = \underbrace{\oint \vec{E}_1 \cdot d\vec{e}} + \oint \vec{E}_2 \cdot d\vec{e}$$

Symmetry $E_2(r=0) = 0$

$$\oint \vec{E} \cdot d\vec{e} = E_2 l = E_2 s = \text{Left hand side}$$

From above,

$$\vec{B} = \frac{\mu_0 I r}{2\pi a^2} \hat{\phi}$$

$$B = B_1 + B_2 + \dots$$

caused by the 1st approx to J_{disp}

$$I_{\text{disp},1} = \epsilon_0 A \frac{dE_1 e^{i\omega t}}{dt}$$

$$I_{\text{disp},1} = \epsilon_0 A i \omega E_1 e^{i\omega t}$$

$$B_1 = \frac{\mu_0 r}{2\pi a^2} \epsilon_0 A i \omega E_1 e^{i\omega t} \quad \text{where } A = \pi a^2$$

$$B_1 = \frac{i\omega}{2} \mu_0 \epsilon_0 r E_1 e^{i\omega t}$$

$$B_1 = i \left(\frac{\omega}{2} \frac{1}{c^2} r \right) E_1 e^{i\omega t}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad \mu_0 \epsilon_0 = \frac{1}{c^2}$$

Faraday's law is now

$$-E_2 s = -\frac{d}{dt} \int B_1 s dr$$

$$E_2 s = \frac{d}{dt} \int \frac{i\omega}{2c^2} E_1 e^{i\omega t} r s dr$$

$$E_2 = -\frac{\omega^2 r^2}{4c^2} E_1 e^{i\omega t}$$

$$E = E_1 + E_2 = E_1 e^{i\omega t} \left(1 - \frac{1}{4} \frac{\omega^2 r^2}{c^2} + \dots \right)$$