Note Title 10/27/2006

10/27/06

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k)\, e^{ikx}\, dk$$

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

insert $F(k)$ integral
into $f(x)$ integral

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\; F(k) \; \right] e^{ikx}\, dx$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(x')\, e^{-ikx'}\, dx' \right] e^{ikx}\, dk$$

x' = dummy integration variable

$\Downarrow$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ik(x-x')} f(x')\,dx'\,dk$$

we can do the k integration so, do this first

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{\left[\int_{-\infty}^{\infty} e^{ik(x-x')}\,dk\right]} f(x')\,dx$$

lets examine this in detail

$$\int_{-\infty}^{\infty} e^{ik(x-x')}\,dk = \lim_{\mu\to\infty}\int_{-\mu}^{\mu} e^{ik(x-x')}\,dk$$

we can do the finite integ

$$\int_{-\mu}^{\mu} e^{ik(x-x')}\,dk = \frac{1}{i(x-x')} e^{ik(x-x')}\Big|_{-\mu}^{\mu}$$

$$= \frac{1}{i(x-x')}\left[e^{i\mu(x-x')} - e^{-i\mu(x-x')} \right]$$

$$= \underbrace{\frac{2\sin(\mu(x-x'))}{(x-x')}}$$

we plotted this function on Wednesday. Let's call it

$$K_\mu(x-x') \equiv 2\,\frac{\sin(\mu(x-x'))}{x-x'}$$

So

$$f(x) = \frac{2}{2\pi} \lim_{\mu \to \infty} \int_{-\infty}^{\infty} f(x') K_\mu (x-x') dx'$$

Let's suppose the limit K_μ as $\mu \to \infty$ exists $\equiv K(x-x')$, then

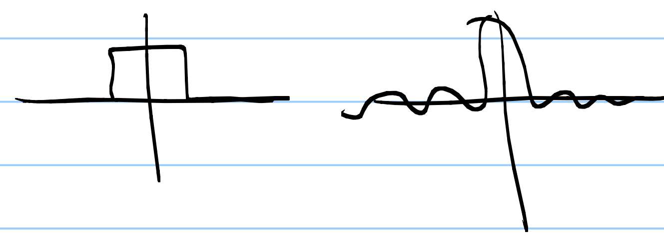
$$\underbrace{f(x) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x') K(x-x') dx'}_{\text{very striking result.}}$$

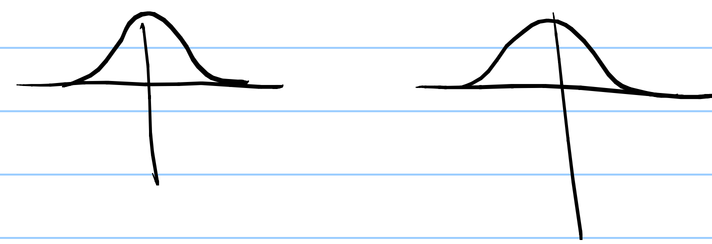
$$\lim_{\mu \to \infty} \frac{K_\mu (x-x')}{\pi} = \delta(x-x')$$

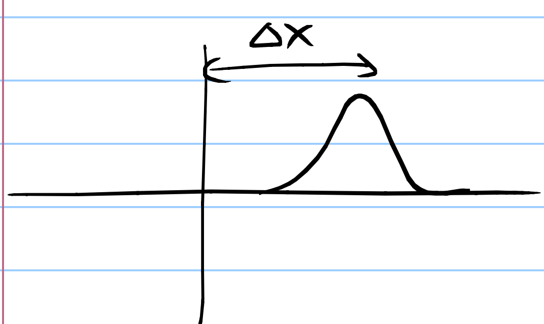
$$f(x) = \int_{-\infty}^{\infty} f(x')\,\delta(x-x')\,dx'$$

$$= f(x)$$

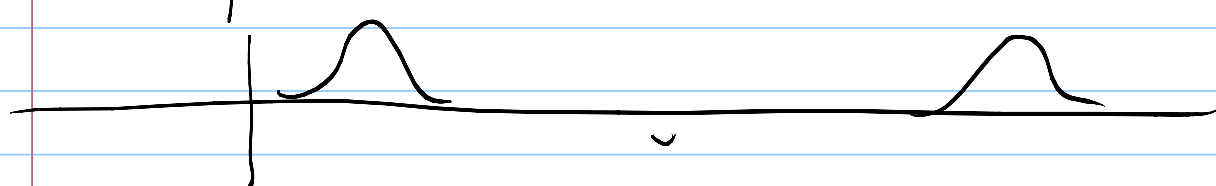
This prooves the invertibility of the Fourier transform.

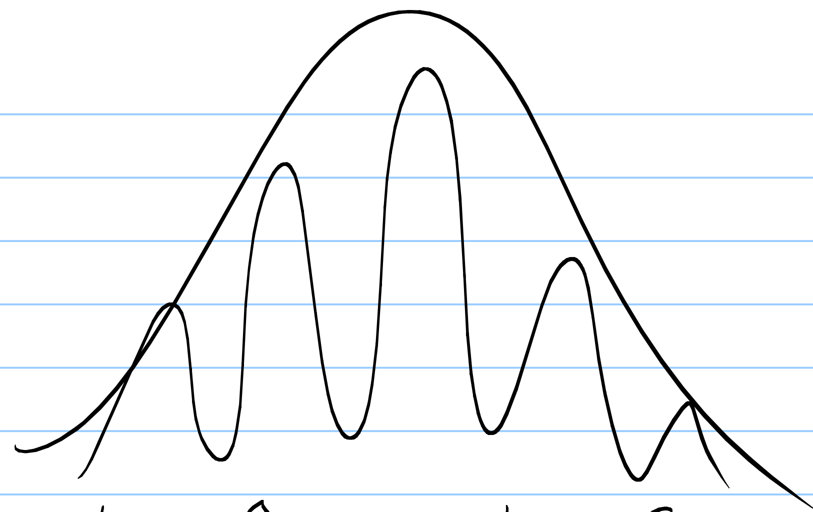
FT of 

FT of 



$e^{i\Delta x k}$ $\quad e^{-(x^2/\sigma^2)}$





spectral interferometry