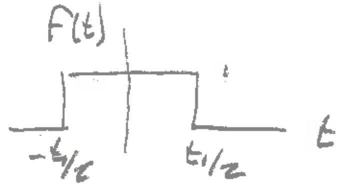
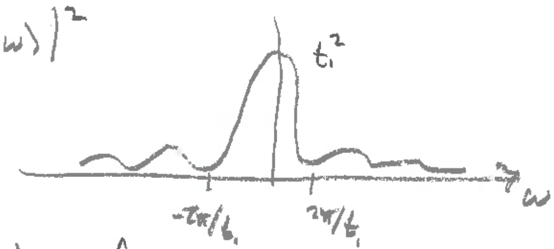
Convolution example

$f_1(t) = \mathrm{rect}\,(t/t_1)$



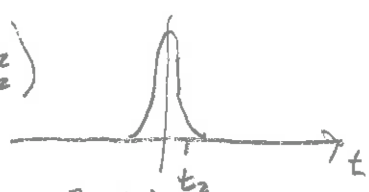
$F_1(t) = t_1\,\mathrm{sinc}\left(\frac{\omega t_1}{2}\right)$



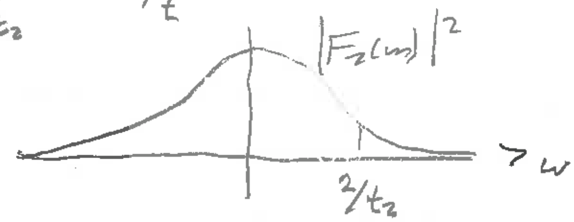
$= 0$ at $\frac{\omega t_1}{2} = \pi$

note fringes extend to high freq.

$f_2(t) = \exp\left(-t^2/t_2^2\right)$



$F_2(\omega) = \sqrt{\pi t_2^2}\,\exp\left(-t_2^2\,\omega^2/4\right)$



$$g_{12}(\tau) = f_1(t) \otimes f_2(t) = \int_{-\infty}^{\infty} f_1(t)\, f_2(\tau - t)\, dt$$

$$= \int_{-t_1/2}^{t_1/2} \exp\left(-\frac{(\tau-t)^2}{t_2^2}\right) dt = \frac{\sqrt{\pi}}{2}\, t_2 \left[\mathrm{Erf}\left(\frac{t_1 - 2\tau}{2t_2}\right) + \mathrm{Erf}\left(\frac{t_1 + 2\tau}{2t_2}\right)\right]$$

softer edges

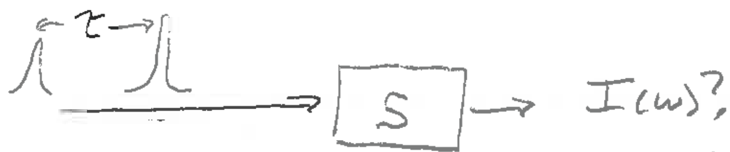
spectrum $G_{12}(\omega) = F_1(\omega)\, F_2(\omega) =$

gaussian cuts out high frequency components

Example - spectral interference

a spectrometer measures $|E(\omega)|^2$

send a pulse pair into spectrometer



$$E_{tot}(t) = E_1(t) + E_2(t) = E(t) + a\,E(t-\tau)$$

FT:

$$E_{tot}(\omega) = E(\omega) + a e^{+i\omega\tau} E(\omega) \quad \text{by shift thm}$$

$$I(\omega) \propto |E_{tot}(\omega)|^2 = |E(\omega)|^2 \left|1 + a e^{i\omega\tau}\right|^2$$

$$= |E(\omega)|^2 \left(1 + |a|^2 + a e^{i\omega\tau} + a^* e^{-i\omega\tau}\right)$$

simplicity - assume $a = a^*$

$$I(\omega) \propto |E(\omega)|^2 \left(1 + a^2 + 2a\cos\omega\tau\right)$$

double pulse $\rightarrow$ interference in the spectrum.

if spectrometer resolution is 1 nm what τ can we see?

fringe period in ω : $\Delta\omega = 2\pi/\tau$ $\qquad \lambda_0 = 500$ nm

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta\omega}{\omega_0} \rightarrow \frac{2\pi}{\tau} = \frac{2\pi c}{\lambda_0^2}\Delta\lambda$$

$$\tau = \frac{\lambda_0^2}{c\Delta\lambda} = 834 \text{ fs}$$

$c\tau \sim 0.25$ mm $\qquad$ corresponds to <u>0.125 mm</u> in interferometer delay line

Linear systems

$$\xrightarrow[f(t)]{} \boxed{S} \xrightarrow[g(t)]{}$$

$g(t) = \hat{S} f(t)$ $\qquad$ $\hat{S}$ = operator for the system

linear if $\hat{S}\left(a_1 f_1(t) + a_2 f_2(t)\right) = a_1 \hat{S}(f_1) + a_2 \hat{S}(f_2)$

i.e. superposition holds

shift invariant if $\hat{S}\left(f(t-t_0)\right) = g(t-t_0)$

in the time domain $\hat{S}$ is time independent.

spatial " $\hat{S}$ is the same for all $\vec{r}$

LSI system is both

Causality:

- in time domain what happens at time t not influenced by later times $> t$

- spatial domain - different

e.g. forward propagation z is like t

no back reflections

> system past can influence present $\qquad$ resonances, delayed respons

General input (any $f(t)$) can always write this

$$g(t) = \hat{\mathcal{L}} f(t) = \hat{\mathcal{L}} \int f(t')\,\delta(t-t')\,dt'$$

$\hat{\mathcal{L}}$ operates on functions of t, not t'

move $\hat{\mathcal{L}}$ inside

$$\therefore\ g(t) = \int f(t') \left(\hat{\mathcal{L}}\,\delta(t-t')\right) dt'$$

$$= \int f(t')\,h(t-t')\,dt'$$

$$= f \otimes h$$

☆ convolve input w/ impulse response to get output.

in frequency space

$$G(\omega) = \mathcal{F}\{f \otimes h\} = \underline{H(\omega)}\,F(\omega)$$

↳ transfer function

<u>implications</u>

system characterization:

1) input ω → measure $A(\omega)$, $\Phi(\omega)$

→ $H(\omega)$ → predict output $g(t)$ for any $f(t)$

2) input $\delta(t)$ measure output $h(t)$ impulse response

→ $H(\omega) = \mathcal{F}\{h(\omega)\}$

applications: filters, signal processing, image processing,

phase effects, control in ultrafast pulses