# When can we assume solutions are complex?

Note Title 9/8/2006

Examples of operators

$$\frac{d}{dx} : f \to f'$$

$$\int : f \to \int f(x)\,dx$$

generic notation $L(f) = g$

$L$ is linear $\Leftrightarrow$ ($\Leftrightarrow$ if and only if) $L(f_1 + f_2) = L(f_1) + L(f_2)$

and $L(cf) = cL(f)$ where

$c$ is a constant.

Usually these are combined:

$$L(cf_1 + f_2) = cL(f_1) + L(f_2)$$

for linear operators

Complicated example:

$$L(x(t)) = \ddot{x} + \gamma\dot{x} + \omega_0^2 x$$

is this a Linear operator?

use def  suppose $x_1(t)$, $x_2(t)$ two diff. functions

$$L(cx_1 + x_2) = (c\ddot{x}_1 + \ddot{x}_2) + \gamma(c\dot{x}_1 + \dot{x}_2) + \omega_0^2(cx_1 + x_2)$$

$$= c\left[\ddot{x}_1 + \gamma\dot{x}_1 + \omega_0^2 x_1\right] + \left[\ddot{x}_2 + \gamma\dot{x}_2 + x_2\right]$$

$$= c\,L(f_1) + L(f_2)$$

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