Diffractive wave propagation

Fresnel propagation

Fraunhofer propagation

Examples of diffraction

3D wave propagation

$$\nabla^2 \mathbf{E} - \frac{{n_j}^2}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = \frac{\partial^2}{\partial z^2}\mathbf{E} + {\nabla_\perp}^2\mathbf{E} - \frac{n(\mathbf{r})^2}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = 0$$

$$\nabla_\perp^2 = \partial_x^2 + \partial_y^2$$

$$\boxed{\nabla_\perp^2 = \frac{1}{r}\partial_r\left(r\,\partial_r\right) + \frac{1}{r^2}\partial_\phi^2}$$

- Note:
  - All linear propagation effects are included in LHS: diffraction, interference, focusing…
  - Previously, we assumed plane waves where transverse derivatives are zero.
- More general examples:
  - Gaussian beams (including high-order)
  - Waveguides
  - Arbitrary propagation
  - Can determine discrete solutions to linear equation (e.g. Gaussian modes, waveguide modes), then express fields in terms of those solutions.

Diffractive propagation

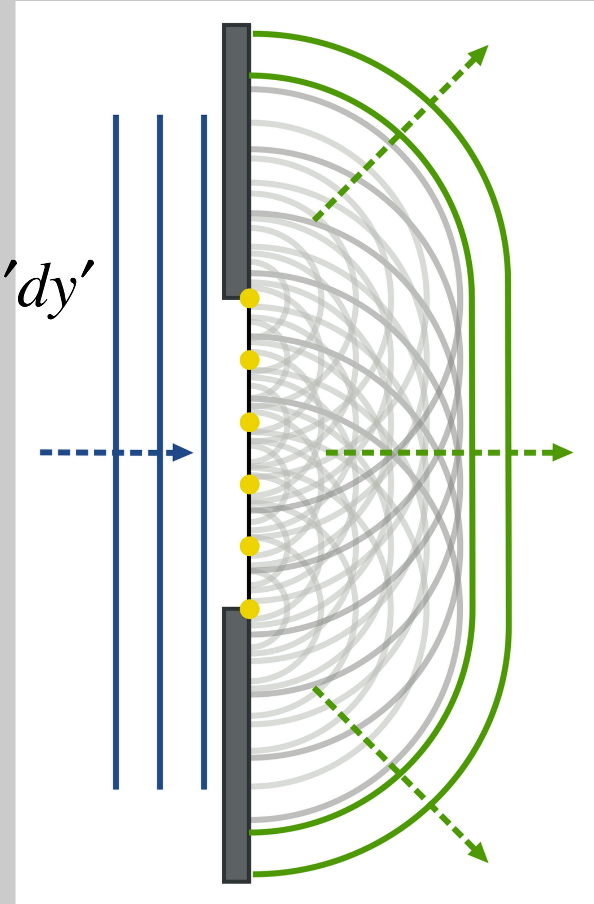
- Huygens' principle:
  - Represent a plane wave as a superposition of source points emitting spherical waves
- Integral representation:

$$E(x,y,z)=\frac{i}{\lambda}\iint \underbrace{E(x',y',z')}_{\text{Field at input plane}}\underbrace{\frac{\exp\left[-ik\left|\mathbf{r}-\mathbf{r}'\right|\right]}{\left|\mathbf{r}-\mathbf{r}'\right|}}_{\text{Spherical wavelet}}\underbrace{\cos\theta}_{\text{Inclination factor}}\,dx'dy'$$

Field at input plane

Spherical wavelet

Inclination factor



This is essentially a convolution of the complex input field with the spherical wavelets, which are the Green's function for the wave equation

Paraxial, slowly-varying approximations

- Assume
  - waves are forward-propagating:
    $\mathbf{E}(\mathbf{r},t) = \mathbf{A}(\mathbf{r}) e^{i(kz-\omega_0 t)} + \text{c.c.}$
  - Refractive index is isotropic

$$\frac{\partial^2}{\partial z^2}\mathbf{A} + 2ik\frac{\partial}{\partial z}\mathbf{A} - k^2\mathbf{A} + \nabla_\perp{}^2\mathbf{A} + \frac{n^2\omega_0{}^2}{c^2}\mathbf{A} = 0$$

  - Fast oscillating carrier terms cancel (blue)
- Slowly-varying envelope: compare red terms
  - Drop 2nd order deriv if $\frac{2\pi}{\lambda}\frac{1}{L}A \gg \frac{1}{L^2}A$
  - This ignores:
    - Changes in z as fast as the wavlength
    - Counterpropagating waves

$$2ik\frac{\partial}{\partial z}\mathbf{A} + \nabla_\perp{}^2\mathbf{A} = 0$$

Fresnel diffraction integral

- Fresnel approximation (near field)
  - Expand the spherical wave in paraxial approximation (in exponential)
  - Let denominator be $\left|\mathbf{r} - \mathbf{r}'\right| \sim z - z' = L \qquad \cos\theta \simeq 1$
  - Input field: $E(x', y', z') = u(x', y', z') e^{+ik(z-z')}$

$$u(x,y,z) = \frac{i}{\lambda L} \iint u(x',y',z') \exp\left[-ik \frac{(x-x')^2 + (y-y')^2}{2L}\right] dx' dy'$$

$$u(x,y,z) = \frac{i}{\lambda L} \mathrm{e}^{-ik\frac{x^2+y^2}{2L}} \iint u(x',y',z') \mathrm{e}^{-ik\frac{x'^2+y'^2}{2L}} \mathrm{e}^{+i\frac{k}{L}(xx'+yy')} dx' dy'$$

Fraunhofer diffraction integral

$$u(x,y,z) = \frac{i}{\lambda L} e^{-ik\frac{x^2+y^2}{2L}} \iint u(x',y',z') e^{-ik\frac{x'^2+y'^2}{2L}} e^{+i\frac{k}{L}(xx'+yy')} dx'dy'$$

- In the "far field", we approximate the sum of paraxial spherical waves as a sum of plane waves
  - Assume field in input plane is confined to a radius *a*
  - If $\frac{ka^2}{2L} = \frac{\pi a^2}{\lambda}\frac{1}{L} << 1$ then we drop quadratic phases.

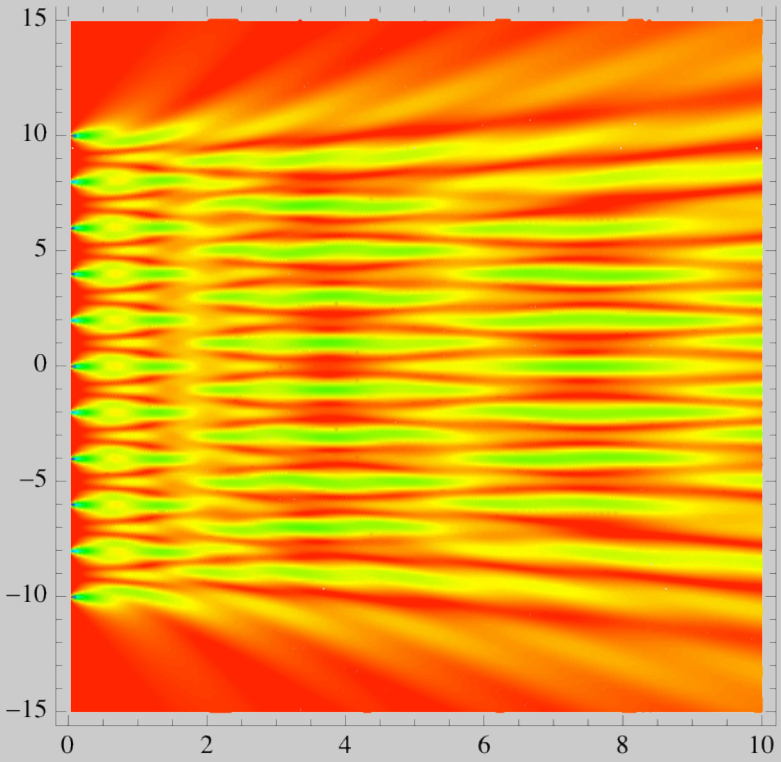
$$u(x,y,z) = \frac{i}{\lambda L} \iint u(x',y',z') \exp\left[+i\left(\frac{kx}{L}x' + \frac{ky}{L}y'\right)\right] dx'dy'$$

  - Result: far field is a Fourier transform of the input field
  - "spatial frequencies" $\beta_x = k\frac{x}{L} = k\sin\theta_x \quad \beta_y = k\frac{y}{L} = k\sin\theta_y$

Example: sum of dipole radiators

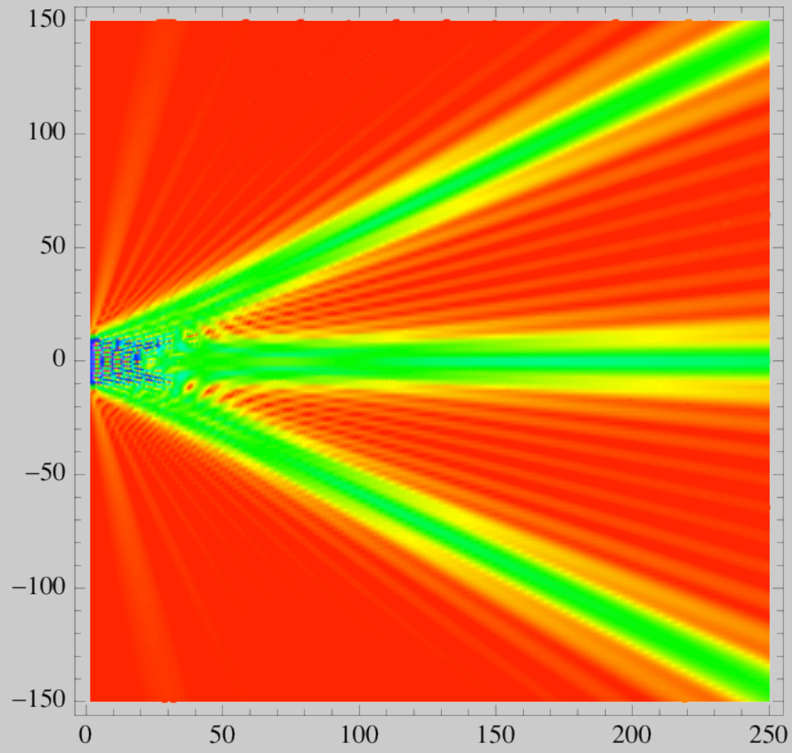
- Add fields from 10 individual sources

Near field



Talbot fringes

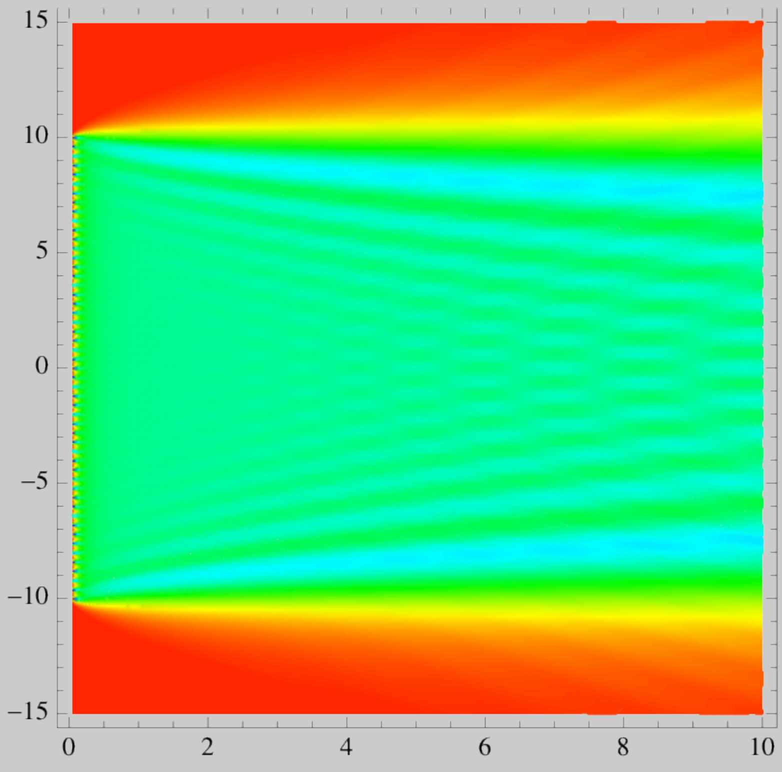
far field



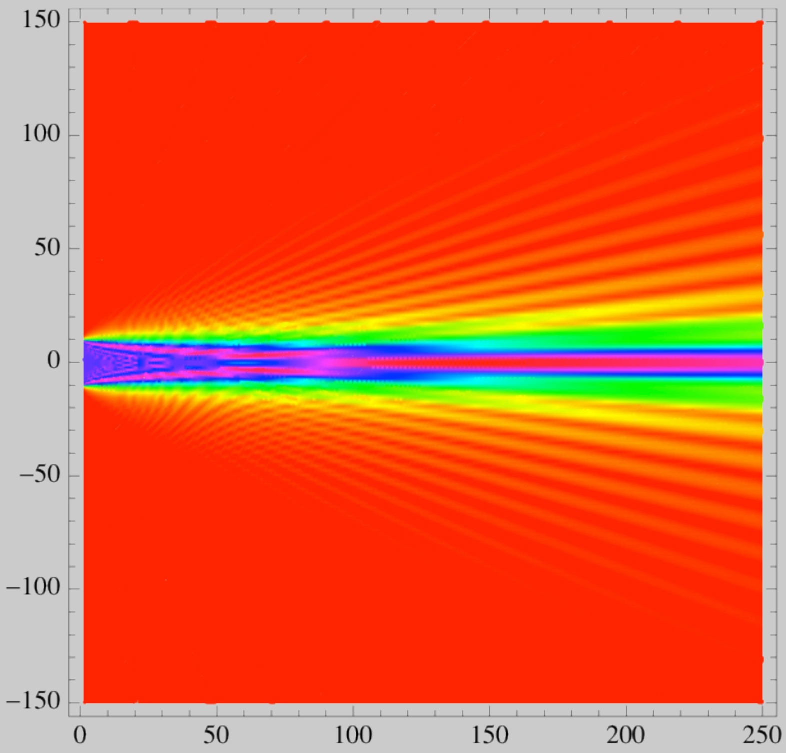
Diffraction grating

High-density of radiators

- Combine 50 sources over same distance



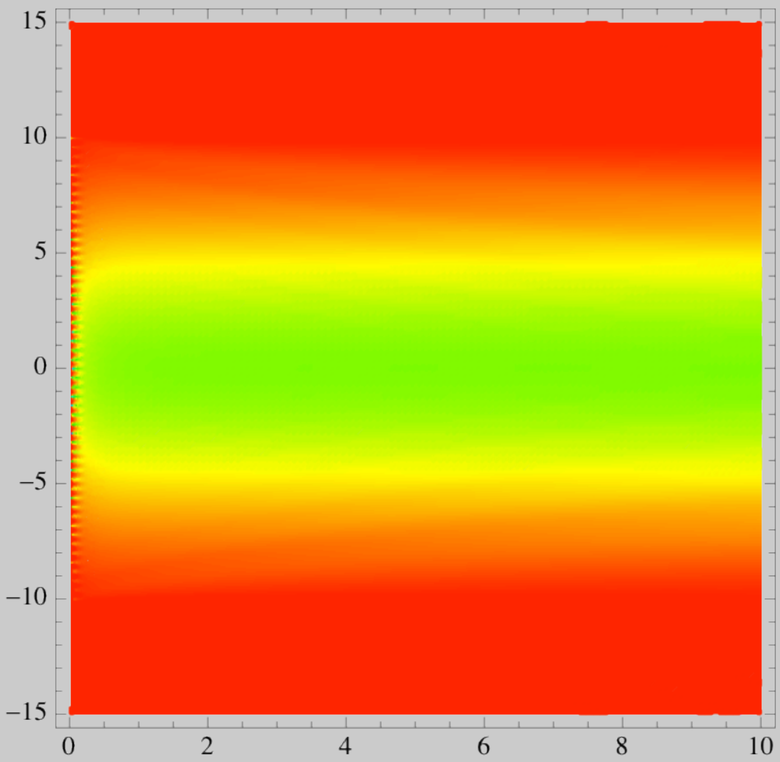
Fresnel zone shows shadow boundary, diffraction fringes

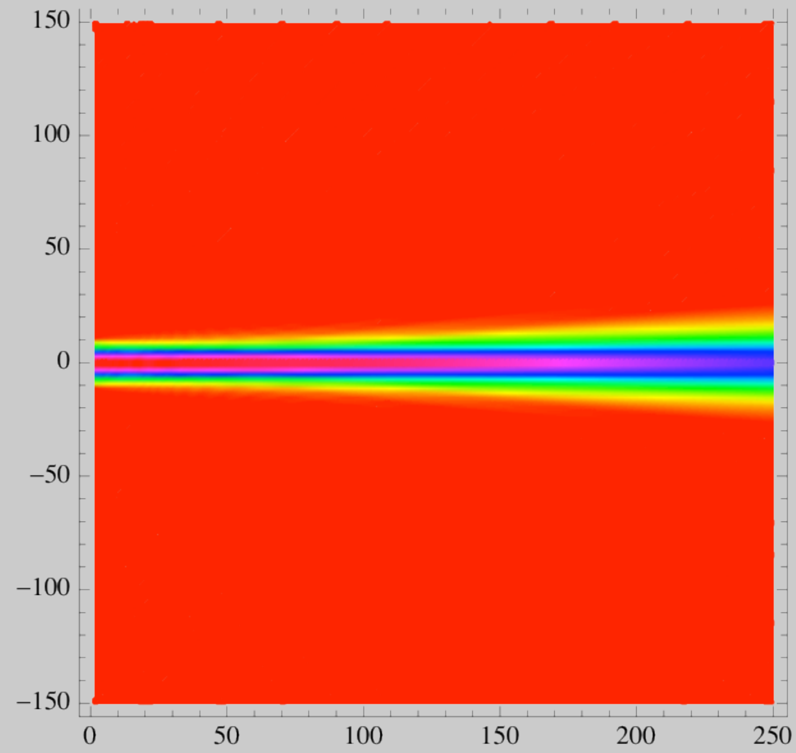


Far field evolves more like a beam, with single-slit diffraction.

High density of radiators, Gaussian envelope

- Gaussian amplitude envelope eliminates diffraction fringes





Beam smoothly spreads out with distance