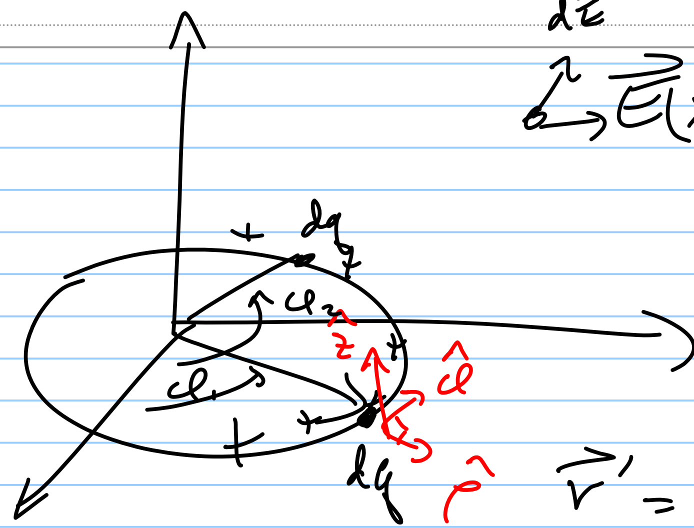


$d\vec{E}$

$$\vec{E}(x,y,z) = \int \frac{k\,dq}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

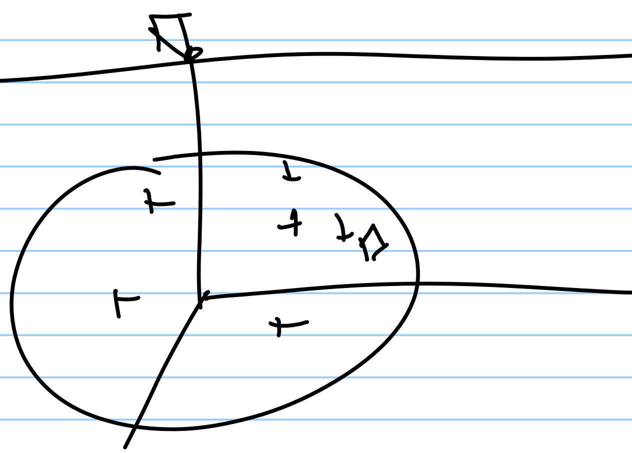
$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' =$$

$$\vec{r}\,' = \underline{R}\,\hat{\rho}' + \underline{\varphi}'\,\hat{\varphi}' + \underline{z}'\,\hat{z}'$$

not constant

$$\hat{\rho}(\varphi)$$

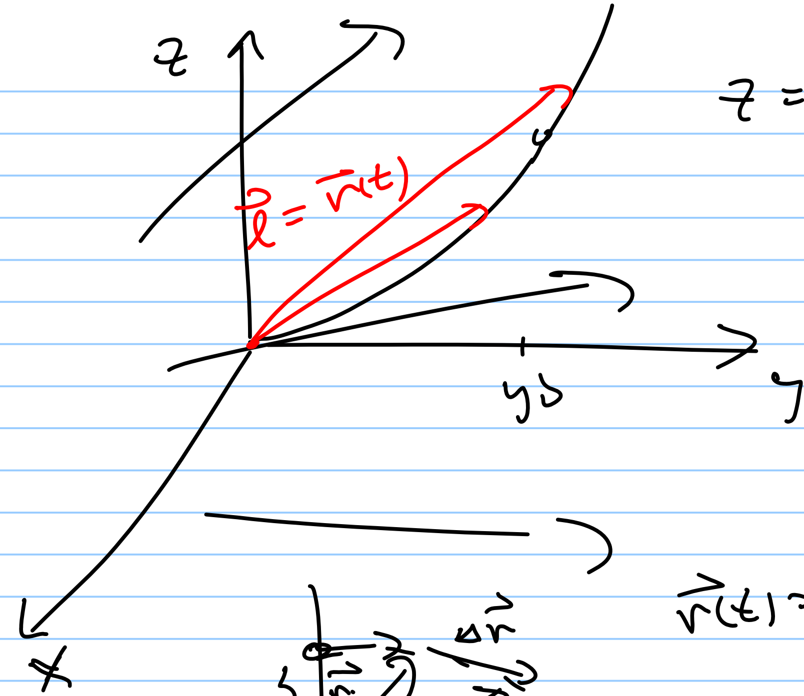
$$\int_0^{2\pi} \hat{\rho}\;d\varphi$$



line ~~integrals~~

$$\int_0 \vec{E}\cdot d\vec{\ell}$$

given



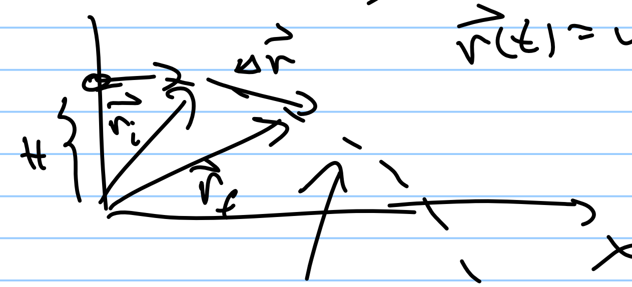
$z = y^2$

$\vec{\ell} = 0\hat{x} + y\hat{y} + y^2\hat{z}$

$d\vec{\ell} = dy\,\hat{y} + 2y\,dy\,\hat{z}$

0 to y_0

$d\vec{r} =$



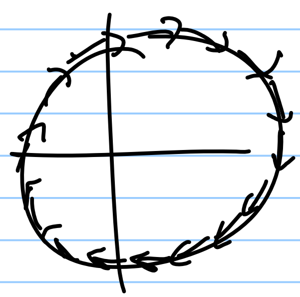
$\vec{r}(t) = v_0 t\,\hat{x} - \frac{1}{2}at^2\hat{y}$

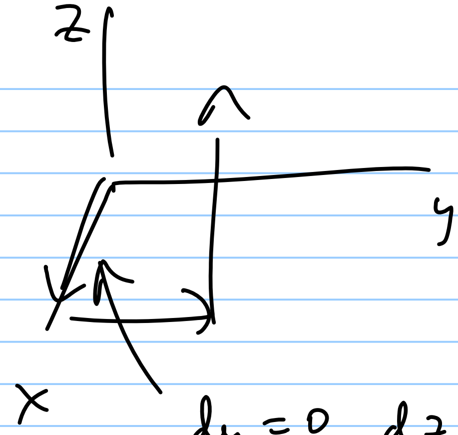
(H-

distance traveled along path

$= \cancel{\oint d\vec{r} = 0}$

$= \int |d\vec{r}|$





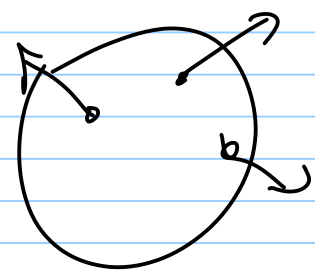
$$d\vec{l} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$dy = 0$ $dz = 0$ $d\vec{l} = dx\,\hat{x}$

flux:

drops enclosed by surface

$$\Phi_{\text{closed surface}} = -\frac{d}{dt} D + S$$



Statics

$$\oint \vec{E}\cdot d\vec{a} = S = \frac{Q_{enc}}{\epsilon_0}$$ Gauss's law

$\vec{E}$

Divergence th.

$$\oint \vec{G}\cdot d\vec{a} = \int \vec{\nabla}\cdot\vec{G}\, d\tau$$

enclosed

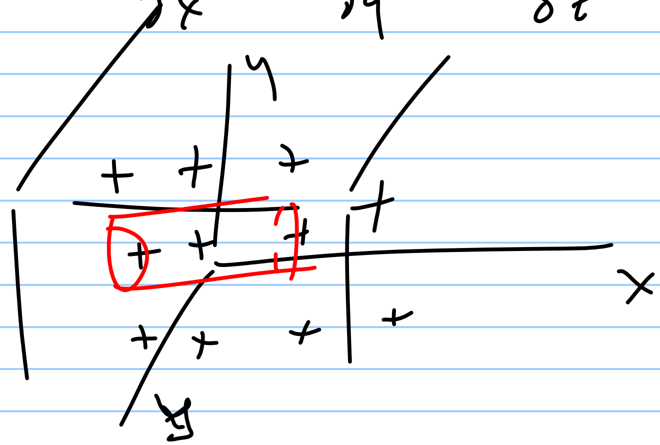
$$\oint \vec{E}\cdot d\vec{a} = \int \vec{\nabla}\cdot\vec{E}\, d\tau = \frac{Q_{encl}}{\epsilon_0} = \frac{\int \rho\, d\tau}{\epsilon_0}$$

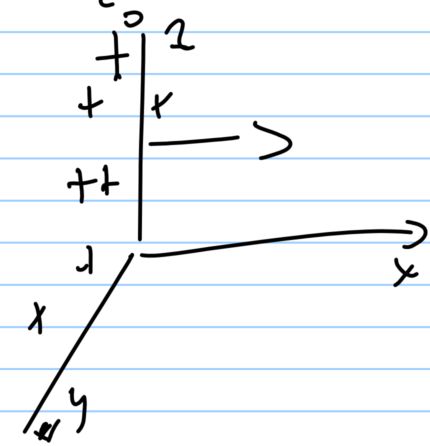
$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$ Maxwell Eqn

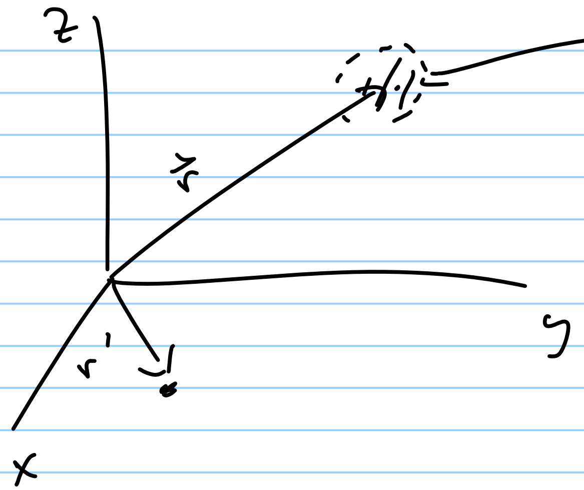
Differential form of $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}$

Ex: given $E_x = -kx$ (k const) find ρ

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = -k = \frac{\rho}{\epsilon_0}$$







$$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{en}}{\epsilon_0} = \phi = \int \vec{\nabla}\cdot\vec{E}\, d\tau$$

$$\vec{\nabla}\cdot\vec{E} = ?$$

$$= \rho/\epsilon_0 = 0$$

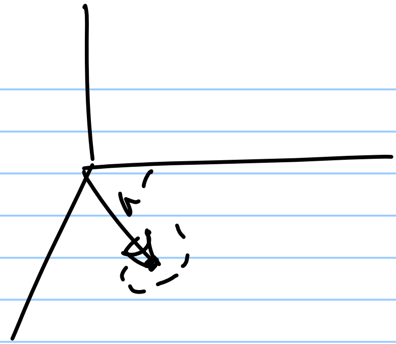
$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{\nabla}\cdot\vec{E} = \vec{\nabla}\cdot\left(\frac{kQ}{r^2}\hat{r}\right) = \vec{\nabla}\cdot \frac{kQ\left[(x-x')\hat{x} + (y-y')\hat{y} + (z-z')\hat{z}\right]}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}$$

$$= \frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z = \frac{1}{[\ ]^{3/2}} - \frac{3}{2}\frac{2(x-x')^2}{[\ ]^{5/2}} + \cdots -$$

$\cancel{\frac{\partial}{\partial x'}}$ not x'

$$= \frac{3}{r^3} - 3\frac{r^2}{r^5} = \phi \qquad \vec{r} = \vec{r} - \vec{r}\,' = \phi$$



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0} \neq 0$$

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$$

for pt charge we need $\delta(\vec{r})$

on x, y, z & not x', y', z'

$$\rho = ?$$

charge $= \int \rho \, d\tau$

$$\frac{Q}{\epsilon_0} = \frac{1}{\epsilon_0} \int \rho \, d\tau$$

wrt $x' y' z'$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r') \, d\tau'}{|r - r'|^2} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{4\pi\epsilon_0}\int \rho(r')\,d\tau'\,\underbrace{\vec{\nabla}\cdot\frac{(\vec{r}-\vec{r}\,')}{|r-r'|^3}}_{4\pi\,\delta(\vec{r}-\vec{r}\,')} = \frac{Q}{\epsilon_0}$$

$\uparrow$ wrt $x\,y\,z$ $\qquad\qquad \uparrow$

$$\frac{1}{\cancel{4\pi}\epsilon_0}\int \rho(r')\,d\tau'\,\cancel{4\pi}\,\delta(\vec{r}-\vec{r}\,') =$$

$$= \frac{1}{\epsilon_0}\iiint \rho\left(\sqrt{x'^2+y'^2+z'^2}\right) dx'\,dy'\,dz'\,\delta(x-x')\,\delta(y-y')\,\delta(z-z')$$

x' integral is $\displaystyle\int_{-\infty}^{\infty} \rho\left(\sqrt{x'^2+y'^2+z'^2}\right)\delta(x-x')\,dx' = \rho\left(\sqrt{x^2+y'^2+z'^2}\right)$

The same for y' & z' so

$$\frac{1}{4\pi\epsilon_0}\int \rho(r')\,d\tau'\,4\pi\,\delta(\vec{r}-\vec{r}\,') = \frac{\rho(\vec{r})}{\epsilon_0}$$

note this is not $\vec{r}-\vec{r}\,'$ nor $\vec{r}\,'$

finally

$$\vec{\nabla} \cdot \vec{E}(x,y,z) = \frac{\rho(x,y,z)}{\epsilon_0}$$

$\vec{r}$

wrt x, y, z