PHGN462 Take-home Midterm II — Name: ______________________

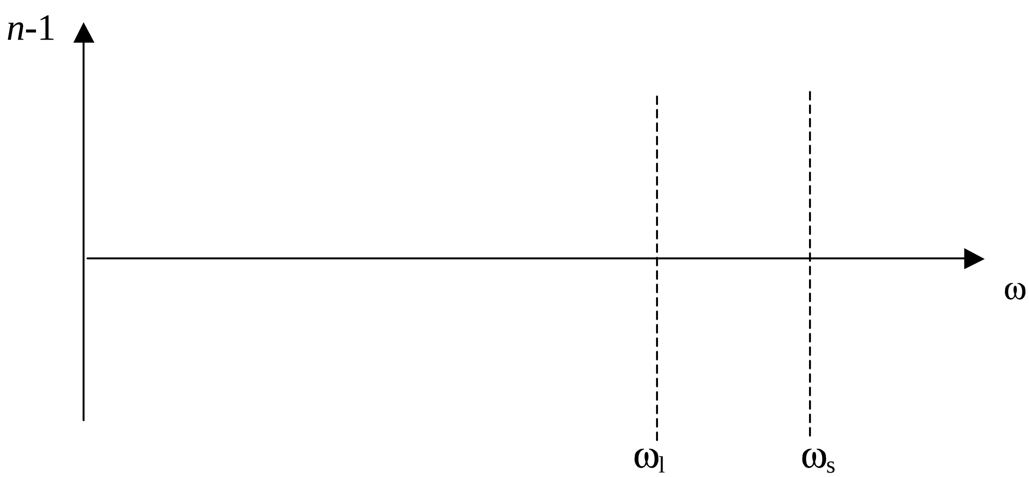
Updated (2 corrections in red) sat 3:25pm — Due 19 Nov 2007 by 5pm

Updated (problems 1d, 3) sun 7:55 am, mon 5:40am

Rules: open books, notes, internet. NO communication with other people. Neat, clear work. Begin each problem on a new page.

1. Suppose we have a gas of a molecule that has a long axis and two, equal short axes. If we apply a static electric field along the x-direction, the molecules will align with the long axis in that direction. We will use a simple, single-resonance model for the response of the molecule to an EM wave. Along the short and long directions of the molecule, the resonance frequencies of the bound electrons are ω_s and ω_l respectively. Assume the damping coefficient is the same for both resonances, and that the oscillator strengths (f_α) are equal to 1. Since the electrons are less tightly bound along the long direction, $\omega_l < \omega_s$.
   a. Consider a plane wave propagating in the z-direction through this gas of molecules aligned in the x-direction. Write two expressions for the complex refractive index (n_x and n_y) experienced by light polarized in the x- and the y-directions. It is not necessary to derive these expressions.

   b. Sketch two curves for the real part of the refractive index that corresponds to these two resonance frequencies.



   c. Suppose we measure the refractive indices at a wavelength of 500nm and we find that n_x-1 = 5x10^{-4} at standard atmospheric pressure, and n_y-1 is 20% lower. The gas is transparent at this wavelength. Estimate values for the two resonance frequencies.
   d. A plane wave of wavelength λ=500nm, with linear polarization oriented at 45^{o} to the x-axis is directed into a gas cell that is 10 cm long. The gas cell has the molecules aligned as described above, and this alignment is unaffected by the input wave. The pressure is brought up from vacuum to a point where the output polarization is observed to rotate by 90^{o} compared to the input polarization. What pressure is required for this to occur? Treat the gas as ideal and at room temperature. Be sure to show how you arrive at your answer.

2. Consider a metallic rectangular waveguide filled with a dielectric with an index of refraction *n*, and with widths *a* and *b* along the *x* and *y* directions, respectively. The propagation direction is in z (see Fig. 7-5 in Heald and Marion).
   a. For the TM modes, write an expression for the longitudinal component of the electric field for the guided modes, $E_z(x,y,z,t)$. Describe the boundary conditions you are applying for this field component. Define the values of the *x*, *y* and *z* components of the k-vector.

   b. Show that the lowest-order mode is TM_{11}.

   c. Now assume the dielectric inside the waveguide is a plasma. Calculate an expression for the z-component for the k-vector, k_z expressed in terms of the number density N_e of the free electrons in the plasma.

   d. Calculate an expression for the cutoff frequencies ω_{mn} for this plasma-filled waveguide, where *m*, *n* are the mode indices for the *x* and *y* directions. Show your work and explain the basis for your reasoning.

   e. Suppose the waveguide is square, with $a = b = 0.01$ mm, and the waveguide is empty (vacuum). What is the cutoff wavelength for the lowest mode?

   f. If the input wavelength to the square waveguide from part e is 7µm, how many modes does the empty waveguide support? At what number density for the plasma will the waveguide only support a single mode?

3. Consider the radiation damping of a moving, oscillating charge. An electron is launched at the origin with a velocity components v_{x0}, v_{z0} into a region where there is a potential $U(x,z) = \frac{1}{2} m\omega_0^2 x^2$.
   a. Calculate the initial (e.g. during the first cycle) time average radiated power, assuming all velocities are much less than *c*, and that the path of the electron during this cycle is not affected by the energy lost to radiation.

   b. Describe how radiation damping affects the motion of the electron: make sketches of the x(t) and z(t). Explain your reasoning.

   c. Sketch the angular distribution of the radiated power in the low velocity limit $v \ll c$, and in the limit where v_{z0}/c is appreciable. Support your answer by relating this physical situation to other examples in the book.