Polarization

given a direction of $\vec{k}$ $\quad \nabla \cdot \vec{E} = 0 \rightarrow \vec{k} \cdot \vec{E} = 0$
allows 2 degrees of freedom for $\vec{E}$ direction.

most general plane wave for $\vec{k} = k_0 \hat{z}$:

$$\vec{E}(\vec{r}, t) = (E_x \hat{x} + E_y \hat{y}) e^{i(k_0 z - \omega t)}$$

amplitude coeff are complex:

$$E_x = E_{x0} e^{i\phi_x}$$

$$E_y = E_{y0} e^{i\phi_y}$$

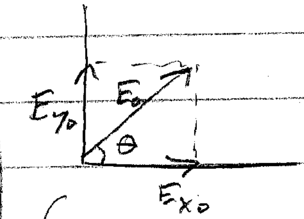
$$\vec{E} = (E_{x0} \hat{x} + E_{y0} e^{i(\phi_y - \phi_x)} \hat{y}) e^{i(k_0 z - \omega t + \phi_x)}$$

relative phase $\phi_y - \phi_x$ determines nature of polarization state.

☆ an overall phase shift doesn't affect pol. state.

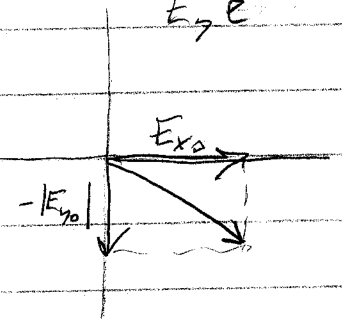
Linear $\quad \phi_x = \phi_y + n\pi \quad n = 0, 1, 2, \ldots$

$n = 0, 2, 4 \ldots$



$n = 1, 3, 5 \ldots$

$E_y e^{i(\phi_y - \phi_x)} \rightarrow -|E_y|$



or more simply,
$\phi_x = \phi_y$
E_x, E_y can be $+/-$

$$\tan\theta = E_{y0}/E_{x0}$$

Jones vector $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$ with E_x, E_y real or: $\vec{E} = E_0 \begin{pmatrix} a \\ b \end{pmatrix} e^{i(k_0 z - \omega t + \phi)}$

w/ $a^2 + b^2 = 1$ $\rightarrow a = \cos\theta$ $b = \sin\theta$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow$ horizontal $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow$ vertical

<u>Polarizer</u>

a linear polarizer passes one direction of pol.

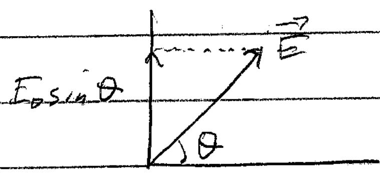
e.g. wire grid, polaroid, birefringent ...

what is transmission of polarizer as fcn. of angle?

$\vec{E} \rightarrow E_0 \begin{pmatrix} a \\ b \end{pmatrix}$

polarizer in <u>vertical</u> direction selects $E_0 \cdot b$ <u>only</u>.

$I(\theta) \propto E_0^2 \sin^2\theta$



$$\vec{E}_{out} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}}_{\text{vertical pol.}} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix}$$

general case: polarizer oriented at angle α

rotate coord. system

Circular polarization

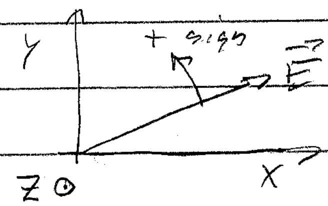
$$E_{x_0} = E_{y_0} \qquad \phi_x - \phi_y = \pm \frac{\pi}{2} \qquad (1/4 \text{ wave})$$

$$e^{\pm i \frac{\pi}{2}} = \pm i$$

$$\vec{E} = E_0 (\hat{x} \pm i \hat{y})\, e^{i(k_0 z - \omega t)}$$

what is time dependence of $\vec{E}$ direction? choose $z = 0$

$$Re(\vec{E}) \sim \hat{x} \cos \omega t \pm \hat{y} \sin \omega t$$



+ sign is called LH cp
("wrong sign")
convention was established very early

Jones vector

$$\vec{E} = E_0 \underbrace{\frac{1}{\sqrt{2}}}_{\text{normalization}} \begin{pmatrix} 1 \\ \pm i \end{pmatrix} e^{i(k_0 z - \omega t)}$$

Question:

a circ. polarized beam with intensity I_0 passes through a linear polarizer, oriented vertically.
What intensity gets through?

$$I_{out} \propto |E_{out}|^2 = |E_y|^2 = \left| \frac{1}{\sqrt{2}} i E_0 \right|^2 = \frac{1}{2} I_0$$

pick projection of $\vec{E}$ onto y axis

Now rotate polarizer → no change.
Projection of circle onto any axis is constant.

Elliptical polarization
- general case.

$$\vec{E} = E_0 (a\hat{x} + b\hat{y}) e^{i(\cancel{kz}-\omega t)}$$

$$|a|^2 + |b|^2 = 1$$

only <u>relative</u> phase is impt
can keep <u>a</u> real if convenient.