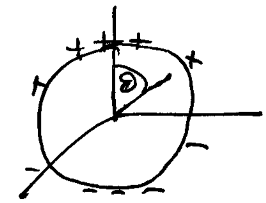
PHGN361 2010 Exam 3: NAME

1. A dipole $\vec{p} = p_0\hat{y}$ is in a non-uniform electric field $\vec{E}(x,y,z)$. Express the force on this dipole in terms of derivatives of $\vec{E}(x,y,z)$.

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E} = \left(p_0 \frac{\partial}{\partial y}\right)\vec{E}(x,y,z)$$

2. Given a sphere of radius R, centered at the origin, which has $\mathbf{P} = 25\hat{z}$ sketch σ_b.



$$\sigma_b = \vec{P}\cdot\hat{n} = 25\,\hat{z}\cdot\hat{r} = 25\cos\theta$$

3. Given a frozen water sphere of radius R, centered at the origin, which has $\sigma_b = \sigma_0\cos^3\theta$ and $\rho_b = 0$ write an integral expression for the voltage inside this sphere.

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{\mathscr{r}} = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma\, da'}{\mathscr{r}}$$

$da' = r'^2 \sin\theta'\, d\theta' d\varphi'$ $\quad \vec{\mathscr{r}} = \vec{r} - \vec{r}'$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{r}' = R\hat{r} = R\sin\theta'\cos\varphi'\hat{x} + \sin\theta'\sin\varphi'\hat{y}, a + \cos\theta'\hat{z}$

4. A wire with charge density λ_{free} has a rubber coating out to $r = R$. Gauss's law yields $\mathbf{D} = \frac{\lambda_f}{2\pi r}$. What is $\vec{E}$ for $r > R$?

$D = \frac{\lambda_f}{2\pi r}$ $\quad D = \epsilon_0 E$ in vacuum $\Rightarrow$ $\epsilon_0 E = \frac{\lambda_f}{2\pi r}$ $\quad \vec{E} = \frac{\lambda_f}{2\pi\epsilon_0 r}\hat{r}$

cylindrical coords $r \Leftrightarrow s$ or ρ

5. Fixed voltage V_0 is applied to a parallel plate capacitor. If the area of the plates is changed by a small amount dA how much charge does the battery move and in what direction does it flow?

CONST V_0 $\quad C = \epsilon_0 A/d$

$Q = CV_0$ $\quad dQ = \left.\frac{\partial Q}{\partial C}\right|_V dC = V_0\, dC = V_0\frac{\epsilon_0}{d}dA$

If dA gets small then dQ gets smaller & battery sucks dQ

6. A constant charge parallel plate capacitor has glass supported inside by a frictionless surface. To prevent the glass from sliding out when the cap is tilted what design changes could be made on the capacitor to reduce the slipping? You can only effect the dimensions of the capacitor. $C = \epsilon_0\chi_e\frac{a}{d}(\frac{w}{2} - x) + \epsilon_0\frac{a}{d}w$ where d is the plate spacing, a and w are the plate dimensions where the glass can move along the w direction a distance x from the center of the cap.

$W_{nc} = \Delta(\cancel{KE}^{\,0} + PE)$ $\quad PE = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} \leftarrow$ constant Q

$$dW_{me} = F_{me}\,dx = d(PE) = \frac{\partial(PE)}{\partial C}\frac{dC}{dx}dx = -\frac{1}{2}\frac{Q^2}{C^2}\frac{dC}{dx}dx$$

Make dW_{me} large & glass stays in cap better $\Rightarrow$

make $\frac{dC}{dx}$ large $= \epsilon_0\chi_e\frac{a}{d} \Rightarrow$ make d small, a large and/or χ_e large