

## Oscillator rate equations

7.2.1

7.3.1

basic idea:

resonator - feedback

- passive loss

gain medium - round trip small signal gain

- saturation at high intensity

S.S. gain < loss

photons last longer in cavity, no oscillation

S.S. gain > loss

above threshold, circulating power ↑

as power ↑, gain saturates

sat gain = loss

steady state

## Rate equations:

account for circulating power

\_\_\_\_\_ 2

\_\_\_\_\_ 1

populations

$$\dot{N}_2 = R_p - B\phi N_1 - N_2/\tau, \quad \dot{\phi} = Y_a B\phi N_1 - \Phi/c_o$$

$\frac{d}{dt} [N_2 - N_1]$	$\xrightarrow{\text{pump rate}}$	$\xrightarrow{\text{stim. emission rate}}$	$\xrightarrow{\text{decay out of level 2}}$	$\xrightarrow{\text{rate of photons generated}}$	$\xrightarrow{\text{cavity loss}}$
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population eqns:

assume 4-level, no population in level 1  $N_1 = 0$

spatially-uniform  $N_2, \Phi, R_p, \dots$

no accounting of  $\lambda$  dependence, homog. broadening.

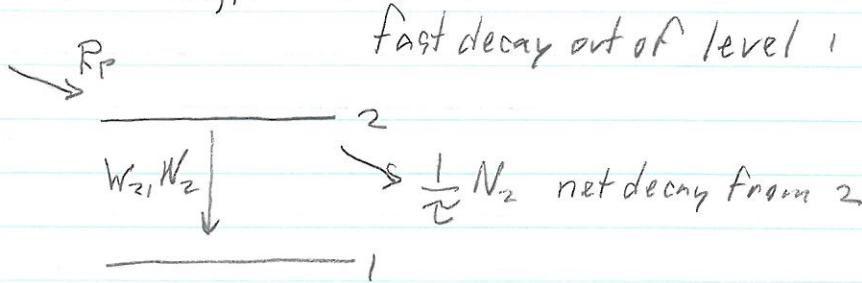
$\tau$  is net decay time out of level 2

- includes non-radiative terms

- includes avg. over many sublevels

Rate equations - simplified 4-level system

$$\Delta N = N_2 - \frac{g_2}{g_1} N_1 \approx N_2 \equiv N$$



write rate eqn:

$$\dot{N} = R_p - W_{21}N - N/2$$

We need another equation for the laser beam.

choose variables:

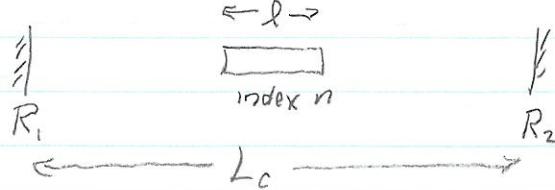
- {  $N$  inversion density
- {  $\phi$  photon number in cavity
- {  $E_{\text{stor.}}$  stored energy density
- {  $P$  circulating power.

we'll use  $N, \phi$

$\therefore$  express stim. emission rate in terms of  $\phi$

$$W_{21} = B_{21}P = \frac{\Omega_{21}}{h\nu} I = \frac{\Omega_{21}}{h\nu} \frac{h\nu \phi}{A_b T_{RT}}$$

space-indep. model:



$A_b$  = area of beam.

$L_i$  = internal loss  
(single pass)

$$T_{RT} = \frac{2L_e}{c} \quad \text{with effective length: } L_e = L_c - l + nl = L_c + (n-1)l$$

$$W_{21} = \frac{\Omega_{21}}{V} \frac{\phi}{(2A_b L_e/c)} = \left(\frac{\Omega_{21} c}{V}\right) \phi \quad V = 2A_b L_e = \text{mode volume.}$$

$$\text{let } B = \frac{\sigma_{z,c} c}{V}$$

$$\frac{dN}{dt} = R_p - B\phi N - \frac{1}{\tau} N.$$

Now get eqn for photon #  $\phi$

stimulated emission adds photons to beam.

$$\frac{d\phi}{dt} = +B\phi N \cdot A_b l \quad \text{S.E. only}$$

$\sim V_a$  = active mode volume

passive cavity  $\rightarrow$  loss of photons.

- output coupling  $T_2$  transmission
- internal loss - scatter, reflections
- mirror absorption.

net effect

$$\frac{d\phi}{dt} = -\frac{1}{\tau_c} \phi \quad \rightarrow \text{exponential decay}$$

$\tau_c$  = photon lifetime.

Photon equation

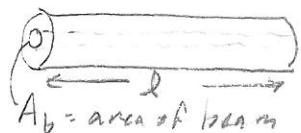
$\Phi$  = total # photons in laser mode.

will find resonators have discrete transverse modes like  
waveguides (recall blackbody calc)

$$W_{12} \equiv B\Phi \quad B\text{-coeff. normalized so that } B\Phi \text{ is rate}$$

$V_a$  = volume of mode in active medium

$$= A_b \cdot l$$



$$V_a N_2 = \# \text{ of inverted atoms}$$

$$\therefore V_a N_2 \cdot W_{12} \text{ rate of stim. emission (gain)}$$

$\tau_c$  = photon lifetime in cavity

$$\text{so that w/o gain, } \dot{\Phi} = -\Phi/\tau_c \Rightarrow \Phi = \Phi_0 e^{-t/\tau_c}$$

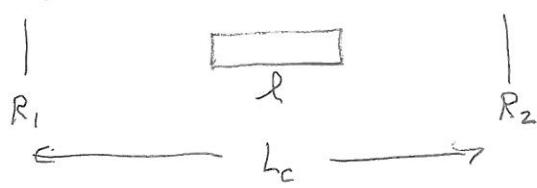
Losses are localized, so this is a continuous approx.

Translation to measurable parameters.

This model is in terms of microscopic parameters - photons, atoms.

Connect to macroscopic:

linear resonator



(per pass)

$L_i$  = internal loss

(scattering, reflections)

$$R_i = (1 - T_i - \alpha_i)$$

effective cavity length = optical path,

$$L_c = L_o + (n-1)l$$

reflectivity transmission

$$\alpha_i = \text{absorptive mirror loss}$$

one round trip

$$\Delta I = I \cdot \underbrace{[R_1 \cdot R_2 \cdot (1 - L_i)^2]}_{\text{loss}} e^{\underbrace{2\sigma N_2 d}_{\text{gain}}} - 1$$

separate mirror transmission (useful) from loss.

$$R_1 R_2 \approx (1 - a - T_1)(1 - a - T_2) \\ \approx (1 - T_1)(1 - T_2)(1 - a)^2$$

Our equation is a difference eqn. - discrete passes  
→ distributed model, differential eqn.

represent losses as logarithmic:

$$\gamma_1 = -\ln(1 - T_1) \quad \text{so that } e^{-\gamma_1} = (1 - T_1)$$

$$\gamma_2 = -\ln(1 - T_2)$$

$$\gamma_i = -\ln[(1 - a)(1 - L_i)]$$

$$\gamma = \gamma_1 + (\gamma_1 - \gamma_2)/2$$

$$\Delta I = I [e^{2(\sigma N_2 d - \gamma)} - 1] \\ \approx I \cdot 2(\sigma N_2 d - \gamma)$$

Assume  $|\sigma N_2 d - \gamma| \ll 1$  slow build-up or loss

$\Delta I$  = change in  $I$  on one round trip

$$\Delta t = \frac{2L_e}{c} = \text{round-trip time} = T$$

$$\frac{\Delta I}{\Delta t} \approx \frac{dI}{dt} = I \left( \frac{2\sigma N_2 d}{2L_e/c} - \frac{2\gamma c}{2L_e} \right) \rightarrow \frac{dI}{dt} = \phi \left( \sigma c L_2 - \frac{\gamma c}{L_e} \right)$$

connect to  $\phi$  eqn:  $\phi \propto I$

$$\rightarrow V_a B N_2 = \frac{\sigma L_e c}{L_e} \quad \boxed{B = \frac{\sigma c}{V}}$$

$$\frac{\text{mode volume}}{V} = \frac{L_e V_a}{l}$$

assume beam size is constant.

photon lifetime in cavity (when no gain)

$$\tau_c = \frac{L_e}{\gamma_c}$$

$$\phi = \frac{I \cdot A_b \cdot T_{RT}}{h \gamma b} = \frac{I \cdot A_b}{h \gamma} \cdot \frac{2L_e}{c} = \frac{2V}{h \gamma c} I$$

$$\text{circulating power } P = I \cdot A_b = \frac{I V}{h_e} = \frac{\phi h \sqrt{c}}{2 h_e} = \frac{\phi h}{T_{RF}}$$

$$\text{output power} = P T_2$$

$$= \frac{\phi h^4 c}{2 L e} \underbrace{(1 - e^{-\gamma_2})}_{\approx \gamma_2}$$

Example :

$$L_1 = 0.01 \quad q = 0.01 \quad T_2 = 0.01$$

$$L_e = 1 \text{ m}$$

$$\rightarrow T = 6.7 \text{ ns} \quad T_c = 133 \text{ mK} \quad \sim 20 \text{ readout fuses}$$

for threshold, need  $G = e^{\sigma_{N_{el}}} > 1.025$

minimized losses: dielectric mirrors

## A/R coatings on Brewster surfaces.

$$\text{Steady-state: } \frac{\partial N_2}{\partial t} = 0 \quad \frac{\partial \dot{N}}{\partial t} = 0$$

otherwise  $\rightarrow$  dynamics: relaxation oscillations

Q-switching  
mode-locking.