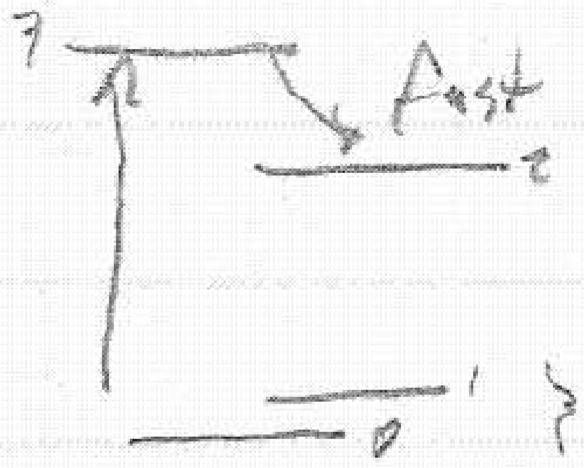
Quasi 3-level system.



} thermally coupled. $\rightarrow N_1$

now keep track of level 1

$$N_1 + N_2 = N_t$$

$$\frac{dN_2}{dt} = R_p - \phi\left(\underset{\substack{\downarrow \\ \text{em.} \sim B_{21}}}{B_e} N_2 - \underset{\text{abs.} \sim B_{12}}{B_a} N_1\right) - \frac{N_2}{\tau} = R_p - \frac{\sigma_e c}{V}\phi(N_2 - fN_1) - \frac{N_2}{\tau}$$

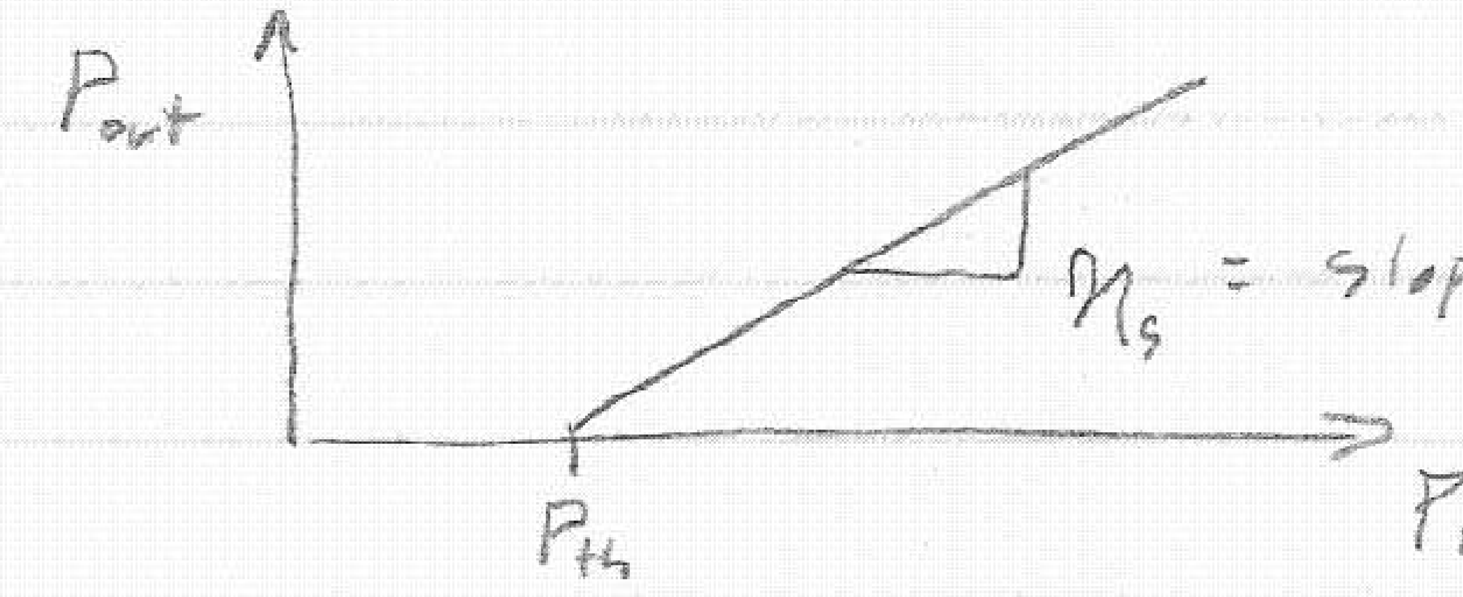
$$\frac{d\phi}{dt} = V_a \phi\left(B_e N_2 - B_a N_1\right) - \frac{\phi}{\tau_c} = \left(\frac{V_a \sigma_e c}{V}\underbrace{(N_2 - fN_1)}_{\text{inversion density } N} - \frac{1}{\tau_c}\right)\phi$$

$$f = \sigma_a / \sigma_e \quad \text{e.g. } g_2/g_1$$

$$\frac{dN}{dt} = \frac{dN_2}{dt} - f\frac{dN_1}{dt} = \frac{dN_2}{dt}(1+f)$$

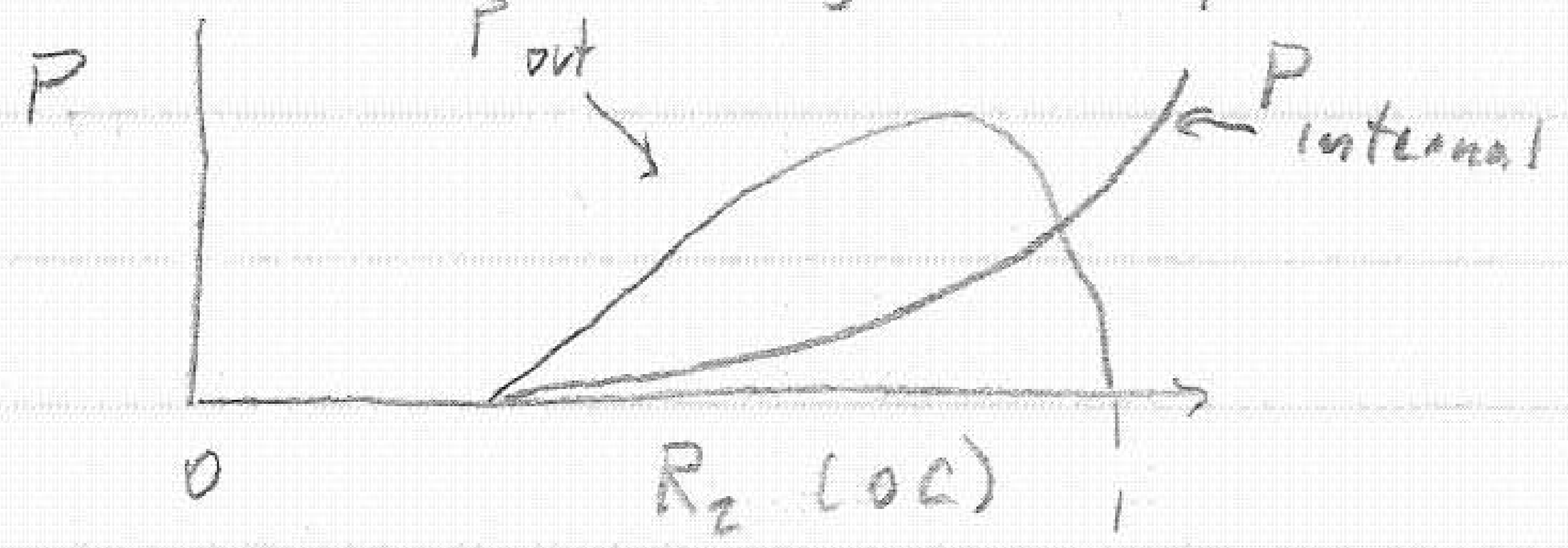
$$= R_p(1+f) - \frac{(\sigma_e + \sigma_a)}{V}\phi N - \left(\frac{fN_t + N}{\tau}\right)$$

CW operation



↳ threshold pump power. - start of lasing.
gain ≳ loss

optimize output coupling - given pump power:



low output coupling → higher internal power, lower output.

Conditions for lasing:

- w/o lasing, but fast τ_{32}, $\quad N_1/\tau_1 = N_2/\tau_{21}$

for any gain, require $\frac{N_2}{N_1} > 1$ so $\tau_{21} > \tau_1$
↓ storage time.

if this is not satisfied,
→ ~ 3 level system, and gain is transient.

critical inversion density:

$$\frac{d\phi}{dt} = 0 = \left(BV_a N_c - \frac{1}{\tau_c}\right)$$

V_a = mode volume

$$B = \sigma_{21}\left(\frac{l}{V_a}\right)\frac{c}{L}$$

$$N_c = 1/BV_a\tau_c =$$

$$BV_a = \frac{\sigma_{21} c l}{L}$$

$$\tau_c = \frac{L}{c\gamma}$$

$$= \frac{L}{\sigma c l}\,\frac{c\gamma}{L} = \frac{\gamma}{\sigma l}$$

Now express slope eff with these factors

$$\eta_s = A_b I_{sat} \frac{\gamma_2}{2} \frac{\eta_p}{\gamma} \frac{1}{A_b I_{sat}} \frac{h\nu_o}{h\nu_{mp}}$$

$$= \eta_p \frac{\gamma_2}{2\gamma} \frac{h\nu_o}{h\nu_{mp}} \frac{A_b}{A}$$

$$= \eta_p \, \eta_c \, \eta_q \, \eta_t$$

η_t → transverse, mode overlap (can't > 1)

η_q → quantum eff

η_c → Output coupling eff.

Quasi-3-level system

rate eqns
$$\frac{\partial \phi}{\partial t} = \left(\frac{V_a \sigma_e c}{V} N - \frac{1}{\tau_c} \right) \phi$$

$$\frac{\partial N}{\partial t} = R_p (1+f) - \frac{(\sigma_e + \sigma_a) c}{V} \phi N - \frac{f N_t + N}{\tau}$$

σ_e emission $\qquad f = \sigma_a / \sigma_e \qquad \frac{V_a}{V} = \frac{l}{L} \qquad \tau_c = \frac{L}{c\gamma}$

σ_a absorption

at threshold, $\frac{\partial \phi}{\partial t} = 0 \rightarrow N_c = \frac{V}{V_a \sigma_e c} \frac{1}{\tau_c} = \frac{\gamma}{\sigma_e l}$

= same as 4-level.

critical pump rate R_{cp} @ $\dot{N} = 0$, $N = N_c$ $\phi = 0$

$$0 = R_{cp} (1+f) - \frac{f N_t + N_c}{\tau}$$

$$R_{cp} = \frac{f N_t + N_c}{(1+f)\tau} \approx \frac{f N_t + N_c}{\tau}$$

typically $f \ll 1$

Yb:YAG $\sigma_a \sim 1.2 \times 10^{-21} \, cm^2$

$\sigma_e \sim 18 \times 10^{-21} \, cm^2$

extra $f N_t$ in numerator → pump harder.

uniform diode pumping

$$R_p = \eta_p \frac{P}{A l\, h\nu_{mp}}$$

$$\rightarrow P_{th} = \frac{(f N_t + N_c)}{(1+f)\tau} \frac{A l\, h\nu_{mp}}{\eta_p} \qquad N_c = \frac{\gamma}{\sigma_e l}$$

$$= \frac{\sigma_a N_t + \gamma/l}{(\sigma_a + \sigma_e)} \frac{h\nu_p}{\tau} \frac{A l}{\eta_p}$$

$$= \frac{\gamma(1 + \sigma_a N_t l/\gamma)}{\eta_p} \frac{h\nu_p}{\tau} \frac{A}{\sigma_a + \sigma_e} \qquad \sigma_a N_t \sim 1/l_{abs}$$

extra

Above threshold: still $\dot{N} = \dot{\phi} = 0$

$$\dot{\phi} = 0 \rightarrow N = N_c \text{ still}$$

$$\dot{N} = 0 \rightarrow$$

$$\phi_0 = \left[-\frac{f N_t + N_c}{\tau} + R_p(1+f)\right] \frac{V}{(\sigma_e + \sigma_a) c N_c}$$

$$= \left(\frac{f N_t + N_e}{\tau}\right)\left(\frac{R_p}{R_{cp}} - 1\right) \frac{V}{N_c c (\sigma_e + \sigma_a)}$$

$$\frac{\left(\frac{\sigma_a}{\sigma_e}\frac{N_t}{N_c} + 1\right) \frac{V_a L}{l}}{c\tau(\sigma_e + \sigma_a)} \rightarrow A_b$$

$$= \frac{A_b \gamma\left(1 + \frac{\sigma_a N_t l}{\gamma}\right)}{\sigma_e + \sigma_a}\left(\frac{P}{P_{th}} - 1\right)\frac{\tau_c}{\tau} \qquad \tau_c = \frac{\gamma_c}{L}$$

output power

$$P_{out} = \frac{\gamma_2 c}{2L} h\nu\, \phi_0 \qquad V = \frac{V_a l}{L}$$

slope eff:

$$\eta_s = \frac{dP_{out}}{dP} = \frac{A_b\left(1 + \frac{\sigma_a N_t l}{\gamma}\right)}{\sigma_e + \sigma_a} \frac{h\nu}{\tau} \frac{\gamma_2}{2} \frac{1}{P_{th}}$$

$\rightarrow$ same as 4-level:

$$\eta_s = \eta_p \frac{\gamma_c}{2\gamma} \frac{h\nu}{h\nu_p} \frac{A_b}{A}$$

even though $h\nu$ is reabsorbed $\rightarrow$ upper state $\rightarrow$ more gain.